Pion production in lepton-nucleon scattering

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Outline of Topics

Motivation

- Long-baseline neutrino oscillation experiments, T2K, MiniBooNE (just finished);
  - Present Experiments $\leftrightarrow E_\nu \sim 1$ GeV;
  - SPP mostly in the first resonance region – $\Delta(1232)$ domain.
- nucleon$\rightarrow \Delta(1232)$ weak transition;
- π^0s from Neutral Current (NC) reactions form a background to $e^\pm$ production, important to account in the measurement of $\nu_\mu \rightarrow \nu_e$ oscillation;
- testing the low-Q^2 QCD;
- $\pi^\pm$ and $\mu^\pm$ produce the same signal in the Cherenkov detector;

SPP Models in νN

- Sato and Lee model, PRC67 (2003) 065201;
- Hernandez et al., PRD76 (2007) 03300) $\leftrightarrow$ Chiral Symmetry;
- B. Serot, X. Zhang arXiv:1011.5913;
- Lalakulich et al. (in reality model of Hernandez et al.) PRD82 (2010) 093001;
- Leitner et al. (isobar model), PRC79, 034601;
- Adler model, Annals Phys. 50 (1968) 189;
- Fogli and Narduli, Nucl. Phys. B160 (1979) 116;
- Rarita-Schwinger formalism for $\Delta(1232)$ excitation;
- Barbero et al. $\leftrightarrow$ Chiral Symmetry. PLB 664, 70.

SPP in Monte Carlo Generators

- **Rein-Sehgal model**: NUANCE, NUET,
- FKR model Relativistic Harmonic Oscillator Quark Model R.P. Feynman, et al., PRD 3, 2706 (1971); F. Ravndal, PRD 4, 1466 ,(1971); F. Ravndal, Lett. Nuovo Cimento, 3 631 (1972) and Nuovo Cimento, 18A 385 (1973); D.Rein and L.M. Sehgal, Annals Phys. 133 (1981) 79, D. Rein, Z. Phys. C 35 (1987) 43.
- Some modifications proposed in: K.M. Graczyk, J.T. Sobczyk, PRD77, 053003 (2008); ibid PRD77, 053001 (2008); C.Berger and L.Sehgal, PRD76 (2007) 113004; K. S. Kuzmin, et al., Mod. Phys. Lett. A 19, 2815 (2004);
- NuWro (Wroclaw Neutrino Generator): Rarita-Schwinger formalism + DIS (Bloom-Gilman duality) Nonresonant background is going to be improved...

Basics of the Model.

- Source: J. Nieves, I. Ruiz Simo and M. J. Vicente Vacas, Phys. Rev. **C83** (2011) 045501
- The tripple-differential cross section for weak pion production on nucleon:

$$\frac{d^3\sigma}{dE' d\Omega'} = \pi G_F^2 \frac{|\mathbf{l}'|}{|\mathbf{l}|} \int \frac{d^3 k}{(2\pi)^3 2E_\pi} L_{\mu\nu} W^{\mu\nu}$$

- Notation: l, l': initial and final lepton 4-momenta, k- pion 4-mometum, $L_{\mu\nu}$, $W_{\mu\nu}$- leptonic and hadronic tensors.

$$L_{\mu\nu} = l_\mu l'_\nu + l'_\nu l_\mu - g_{\mu\nu} l \cdot l' + i\epsilon_{\mu\nu\alpha\beta} l'^\alpha l^\beta$$

$$W^{\mu\nu} = \frac{1}{4M} \overline{\sum_s} \iint \frac{d^3 p'}{(2\pi)^3} \frac{1}{E'_N} \delta^{(4)}(p' + k - p - q) \left\langle N'\pi \left| j^\mu_{cc+}(0) \right| N \right\rangle \left\langle N'\pi \left| j^\nu_{cc+}(0) \right| N \right\rangle^*$$

Basics of the Model, Δ Resonance.

- Main challenge: how to construct a proper $\langle N'\pi | j^{\mu}_{cc+}(0) | N\rangle$ transition current?
- Resonant pion production through Δ-isobar. Δ excitation vertex:

$$\begin{aligned}
\Gamma^{\alpha\mu}(p,q) &= \left[V^{\alpha\mu}_{3/2} - A^{\alpha\mu}_{3/2}\right]\gamma^5 = \left[\frac{C_3^V}{M}(g^{\alpha\mu}\not{q} - q^{\alpha}\gamma^{\mu}) + \right. \\
&+ \frac{C_4^V}{M^2}(g^{\alpha\mu} q\cdot(p+q) - q^{\alpha}(p+q)^{\mu}) + \frac{C_5^V}{M^2}(g^{\alpha\mu} q\cdot p - q^{\alpha} p^{\mu}) + \\
&+ \left. g^{\alpha\mu} C_6^V\right]\gamma^5 + \left[\frac{C_3^A}{M}(g^{\alpha\mu}\not{q} - q^{\alpha}\gamma^{\mu}) + \right. \\
&+ \left. \frac{C_4^A}{M^2}(g^{\alpha\mu} q\cdot(p+q) - q^{\alpha}(p+q)^{\mu}) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2} q^{\alpha} q^{\mu}\right]
\end{aligned}$$

- C_i: set of vector and axial form factors

Basics of the Model, Axial Current for $N-\Delta$ transition

- SU(6) quark model

$$C_5^V(Q^2)=0,\quad C_4^V(Q^2)=-\frac{M}{W}C_3^V(Q^2); \tag{1}$$

- Magnetic Dominance model
- Lalakulich: more detailed analysis based on the multipole decomposition, fit of helicity amplitudes.

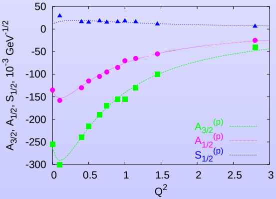


Figure: taken from O. Lalakulich and E. Paschos, Acta Phys.Polon.B37:2311-2319,2006, Max Born XX, Wroclaw

Basics of the Model, Δ Resonance

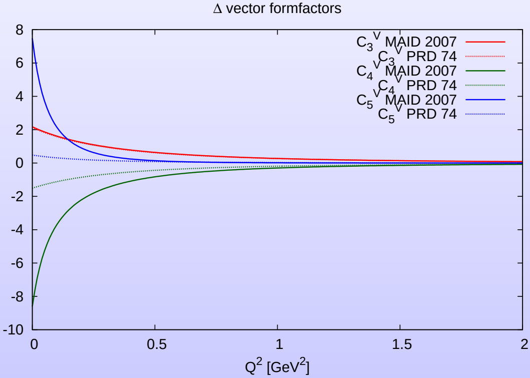
- From CVC $C_6^V = 0$. The vector part in PRD 76 from Lalakulich, Paschos and Piranishvili Phys. Rev. **D 74**, 014009 (2006)

$$
\begin{aligned}
C_3^V(Q^2) &= \frac{2.13}{(1+Q^2/M_v^2)^2}\frac{1}{1+Q^2/4M_v^2} \\
C_4^V(Q^2) &= \frac{-1.51}{(1+Q^2/M_v^2)^2}\frac{1}{1+Q^2/4M_v^2} \\
C_5^V(Q^2) &= \frac{0.48}{(1+Q^2/M_v^2)^2}\frac{1}{1+Q^2/0.776M_v^2}
\end{aligned}
$$

- More recent helicity amplitudes analysis also available (like the MAID 2007 parametrizations).

Basics of the Model, Δ Resonance.

- MAID 2007 vs. Lalakulich et.al:



- Leading contribution: C_3^V. Differences in the others: unimportant. We use Lalakulichet al.

Basics of the Model, Axial Current for $N - \Delta$ transition

- C_5^A:
  - an analog of the isovector axial form factor F_A of the nucleon;
  - PCAC relates $C_5^A(0)$ value with the strong $g_{\pi N\Delta}$ coupling constant Goldberger-Treiman off diagonal relation

$$C_5^A(Q^2) = \frac{g_{\pi N\Delta}}{\sqrt{2}} = 1.15 \pm 0.01 \quad (2)$$

  - $C_5^A(Q^2)$ gives dominant contribution to the cross section at low Q^2
  - **Gives a dominant contribution to forward scattering and for Coherant Pion Production (Hernandez et al., PRD82:077303,2010)** $\rightarrow$ $C_5^A(0)$ value is of great importance...

Basics of the Model, Axial Current for $N - \Delta$ transition

- Parity violating electron scattering measurements may give some information about C5A/C3V (Mukhopadhyay et al. Nucl. Phys. A633 (1998), 481)
- Neutrino Cross Section Data main source of the information about the axial transition form factors.
- neutrino-deuteron scattering data, ANL, BNL data
- After imposing Adler relation C_5^A form factor fully determines the axial contribution to the cross sections, and the low-Q^2 cross section.

Basics of the Model, Axial Current for $N - \Delta$ transition

$C_5^A(0)$:

$C_5^A(0)$	Quark Model
0.97	Ravndal (FKR $\rightarrow$ Rein-Seghal Model)
0.83	Le Yaouanc et al. PRD15, 2447 (1977)
1.17	Hemmert et al. PRD51, 158 (1995)
1.06	M. Bayer Hab.. Thesis
0.87	PRD51, 158 (1995)
1.5	PLB553, 51 (2003)
0.93	Hernandey et al. PRC75, 065203 (2007)
	Empirical Models
From 1.0, to 1.39	

Basics of the Model, Axial Current for $N - \Delta$ transition

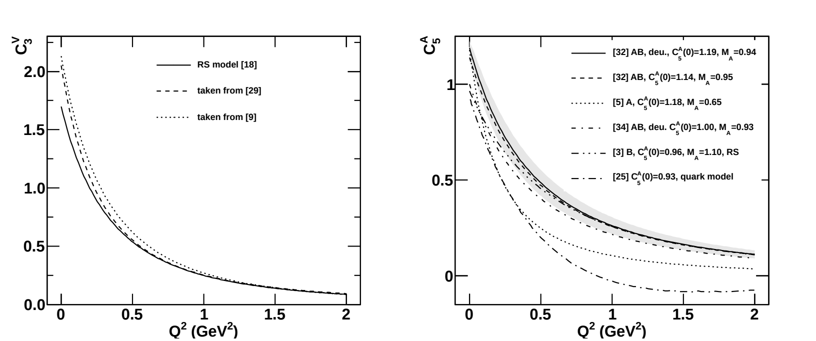


Figure: taken from K.M.G., AIP Conf. Proc. 1405, (2011) 134

Basics of the Model, Axial Current for $N-\Delta$ transition

- C_6^A:
  - an analog of the induced pseudoscalar form factor of the nucleon

$$C_6^A(Q^2) = \frac{M^2}{m_\pi^2 + Q^2} C_5^A(Q^2) \tag{3}$$

  - usually related with C_5^A via PCAC relation
- C_3^A:
  - axial counterpart of the electric quadropole (E2) transition form factor GE2
  - usually it is assumed that $C_3^A = 0$, constraint supported by lattice QCD calculations (see C. Alexandrou et al.), chiral quark model computations (see: D. Barquilla-Cano et al.) and perturbative CFT (see: L.S. Geng et al.. Phys. Rev. D78, 014011 (2008))

Basics of the Model, Axial Current for $N - \Delta$ transition

- C_4^A :
  - an axial counterpart of the charge quadropole transition form factor G_C^2
  - Related with C_5^A (in the SU(6) symmetry limit)
  - Negligible according to the lattice QCD (Alexandrou et al.)

Basics of the Model, Axial Current for $N - \Delta$ transition

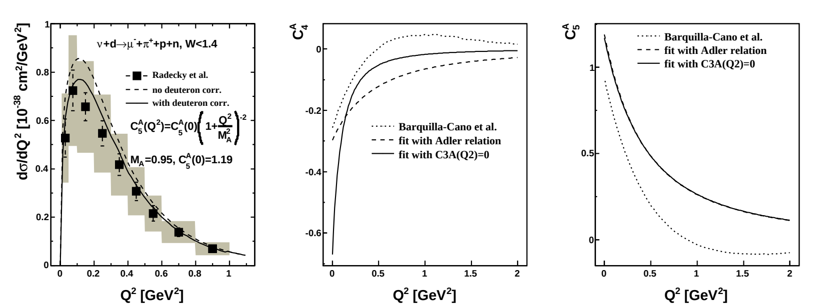


Figure: taken from K.M.G., PoS (EPS-HEP 2009) 286

Basics of the Model, Δ Resonance

- Axial part: connected strictly to $\Delta \to N\pi$ decay through PCAC. The decay vertex:
$$\mathcal{L}_{\pi N\Delta} = \frac{f^*}{m_\pi}\overline{\psi}_\mu \mathbf{T}^\dagger(\partial^\mu \phi)\psi + h.c.$$
- $\overline{\psi}_\mu$- Rarita-Schwinger field, $\mathbf{T}^\dagger$- isospin 1/2→3/2 transition matrix. Value of $f^* = 2.16$ fixed by $\Gamma_\Delta(M_\Delta^2) = 118[MeV]$.
- Default value of $C_5^A(0)$ from the Goldberger-Treiman relation. General form, a dipole:
$$C_5^A(0) = \sqrt{\frac{2}{3}}\frac{f_\pi}{m_\pi}f^* = 1.19;\ \ C_5^A(Q^2) = \frac{C_5^A(0)}{(1+Q2/M_A^2)^2}$$
- Both $C_5^A(0)$ and M_A analysis dependent: fit.
- other axial formfactors:
$$C_3^A = 0;\ \ C_4^A = -1/4C_5^A(Adler\ Model);\ \ C_6^A = C_5^A M^2/(Q^2+m_\pi^2)$$

Basics of the Model, Nonresonant Background

- Nonresonant background from the $SU(2)$ nonlinear σ-model Lagrangian:

$$\mathcal{L}_{\pi N} = i\overline{\psi}\gamma^{\mu}\left[\partial_{\mu} + V_{\mu}\right]\psi - M\overline{\psi}\psi + g_A\overline{\psi}\gamma^{\mu}\gamma^{5}A_{\mu}\psi + \frac{1}{2}\mathrm{Tr}\left[\partial_{\mu}U^{\dagger}\partial^{\mu}U\right]$$

- with a notation for nucleon isodoublets:

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}$$

Basics of the Model, Nonresonant Background

- The vector and axial vector fields:

$$V_\mu = \frac{1}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right)$$
$$A_\mu = \frac{i}{2}\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right)$$

- π: Goldstone bosons related to the $SU(2)_V \times SU(2)_A$ spontaneous chiral symmetry breaking, π hidden in the matrix field

$$U = \frac{f_\pi}{\sqrt{2}} e^{i\boldsymbol{\tau}\cdot\boldsymbol{\phi}/f_\pi} = \frac{f_\pi}{\sqrt{2}}|\xi|^2$$

- Basically: effective field theory with the same chiral symmetry breaking pattern, as QCD.

Basics of the Model, Nonresonant Background

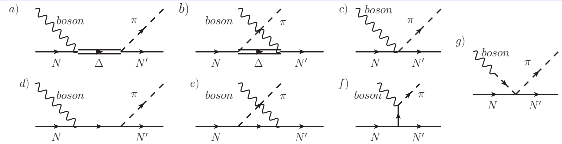
- From the infinitesimal field transformations: lowest order pionic currrents

$$
\begin{aligned}
\mathbf{V}^{\mu} &= \boldsymbol{\phi}\times\partial^{\mu}\boldsymbol{\phi} + \overline{\psi}\frac{\boldsymbol{\tau}}{2}\gamma^{\mu}\psi + \frac{g_A}{2f_\pi}\overline{\psi}\gamma^{\mu}\gamma^{5}(\boldsymbol{\phi}\times\boldsymbol{\tau})\psi + \\
&- \frac{1}{4f_\pi^2}\overline{\psi}\gamma^{\mu}\left[\boldsymbol{\tau}\boldsymbol{\phi}^2 - \boldsymbol{\phi}(\boldsymbol{\tau}\cdot\boldsymbol{\phi})\right]\psi - \frac{\boldsymbol{\phi}^2}{3f_\pi^2}(\boldsymbol{\phi}\times\partial^{\mu}\boldsymbol{\phi}) + \mathcal{O}(1/f_\pi^3) \\
\mathbf{A}^{\mu} &= f_\pi\partial^{\mu}\boldsymbol{\phi} + g_A\overline{\psi}\gamma^{\mu}\gamma^{5}\frac{\boldsymbol{\tau}}{2}\psi + \frac{1}{2f_\pi}\overline{\psi}\gamma^{\mu}(\boldsymbol{\phi}\times\boldsymbol{\tau})\psi + \frac{2}{3f_\pi}(\boldsymbol{\phi}(\boldsymbol{\phi}\cdot\partial^{\mu}\boldsymbol{\phi}) + \\
&- \partial^{\mu}\boldsymbol{\phi}\boldsymbol{\phi}^2) - \frac{g_A}{4f_\pi^2}\overline{\psi}\gamma^{\mu}\gamma^{5}\left[\boldsymbol{\tau}\boldsymbol{\phi}^2 - \boldsymbol{\phi}(\boldsymbol{\tau}\cdot\boldsymbol{\phi})\right]\psi + \mathcal{O}(1/f_\pi^3)
\end{aligned}
$$

- Consistent way of generating necessary CC and EM pion-nucleon system interactions.

Basics of the Model

- All diagrams in this model:



- *a*) "Delta Pole", *b*) "Crossed Delta Pole", *c*) "Contact Term", *d*) "Nucleon Pole", *e*) "Crossed Nucleon Pole", *f*) "Pion In Flight", *h*) "Pion Pole".
- Nucleon current operator ($F_i^V = F_i^p - F_i^n$):

$$\Gamma_N^\mu = V_N^\mu - A_N^\mu = \gamma^\mu F_1^V(Q^2) + i\sigma^{\mu\alpha} q_\alpha \frac{F_V^2(Q^2)}{2M} + G_A(Q^2)\left(\gamma^\mu + \frac{\not{q}\, q^\mu}{q^2 - m_\pi^2}\right)\gamma^5$$

- Additional backround form-factors F_{PIF}^V and F_{PP}^V: fixed from CVC to F_1^V.

Basics of the Model, Formula:

$$\left\langle N'\pi \left| j^\mu_{cc+}(0) \right| N \right\rangle = \overline{u}_{s'}(\mathbf{p}')s^\mu u_s(\mathbf{p})$$

$$s^\mu_{\Delta P} = iC_{\Delta P}\frac{f^*}{m_\pi}\sqrt{3}\cos\Theta_C \frac{k^\alpha P_{\alpha\beta}(p+q)\Gamma^{\beta\mu}(p,q)}{(p+q)^2 - M_\Delta^2 + iM_\Delta\Gamma((p+q)^2)}$$

$$s^\mu_{\Delta PC} = iC_{\Delta PC}\frac{f^*}{m_\pi}\frac{1}{\sqrt{3}}\cos\Theta_C \frac{\gamma^0\left[\Gamma^{\alpha\mu}(p-k,-q)\right]^\dagger \gamma^0 P_{\alpha\beta}(p-k)k^\beta}{(p-k)^2 - M_\Delta^2 + iM_\Delta\Gamma((p-k)^2)}$$

$$s^\mu_{NP} = -iC_{NP}\frac{g_A}{\sqrt{2}f_\pi}\cos\Theta_C \frac{\not{k}\gamma^5(\not{p}+\not{q}+M)}{(p+q)^2 - M^2 + i\epsilon}\left[V^\mu(q) - A^\mu(q)\right]$$

$$s^\mu_{NPC} = -iC_{NPC}\frac{g_A}{\sqrt{2}f_\pi}\cos\Theta_C \left[V^\mu(q) - A^\mu(q)\right]\frac{(\not{p}-\not{k}+M)\not{k}\gamma^5}{(p-k)^2 - M^2 + i\epsilon}$$

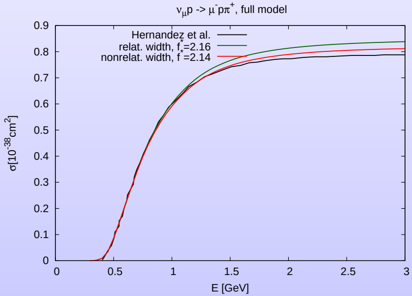
$$s^\mu_{CT} = -iC_{CT}\frac{1}{\sqrt{2}f_\pi}\cos\Theta_C\gamma^\mu\left[g_A F^V_{CT}(q^2)\gamma^5 - F_\rho((q-k)^2)\right]$$

$$s^\mu_{PIF} = -iC_{PIF}\frac{g_A}{\sqrt{2}f_\pi}\cos\Theta_C F^V_{PIF}(q^2)\frac{2M(2k^\mu - q)\gamma^5}{(k-q)^2 - m_\pi^2}$$

$$s^\mu_{PP} = -iC_{PP}\frac{1}{\sqrt{2}f_\pi}\cos\Theta_C F_\rho((q-k)^2)\frac{q^\mu\not{q}}{q^2 - m_\pi^2}$$

Tests of the Model

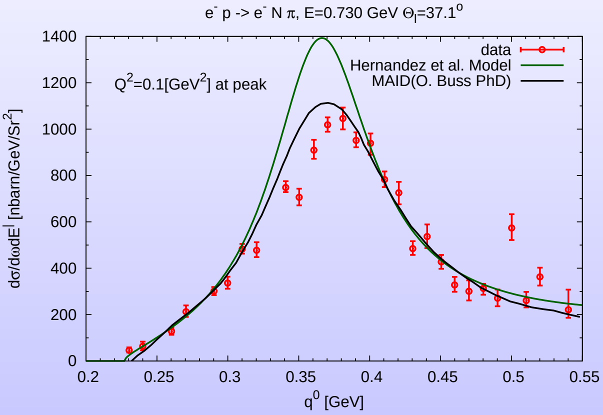
- In PRD 76 nonrelativistic Δ width and slightly different coupling ($f^* = 2.14$).
- Comparison of both widths and couplings directly to PRD 76:



- Small, model dependent channges.

Tests of the Model

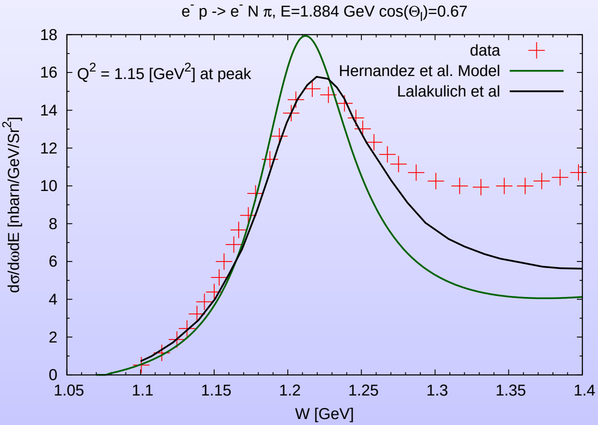
- Electroproduction, electron data and MAID:



- Vector part has serious flaws, too high at the peak.

Tests of the Model

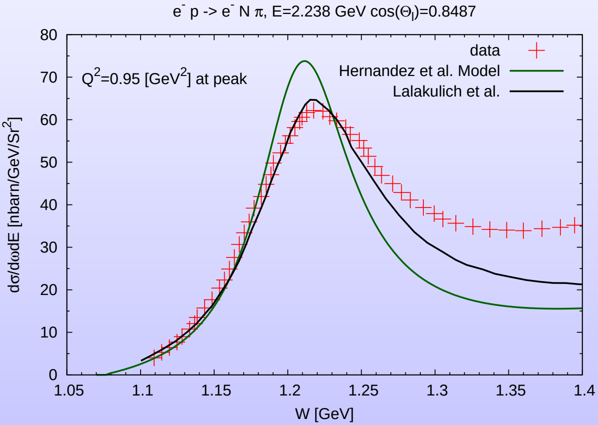
- Electroproduction, electron data and Phys. Rev. D **82** (2010) 093001



- Vector part has serious flaws, too high at the peak.

Tests of the Model

- Electroproduction, electron data and Phys. Rev. D **82** (2010) 093001:



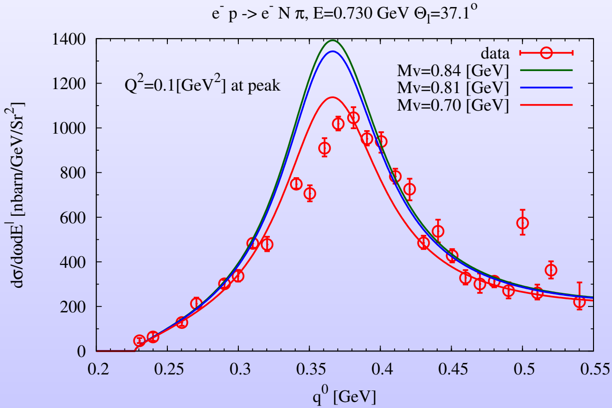
- Vector part has serious flaws, too high at the peak.

Vector Part Problem

- Vector part visibly off the data!
- Probable mismatch between form-factors and the model of Δ and background $+ \pi N$ decay width.
- Solution: parametrization of the vector part uncertainty. Example solution: changes in the vector mass of Δ in the parametrization of formfactors.

Vector Part Problem

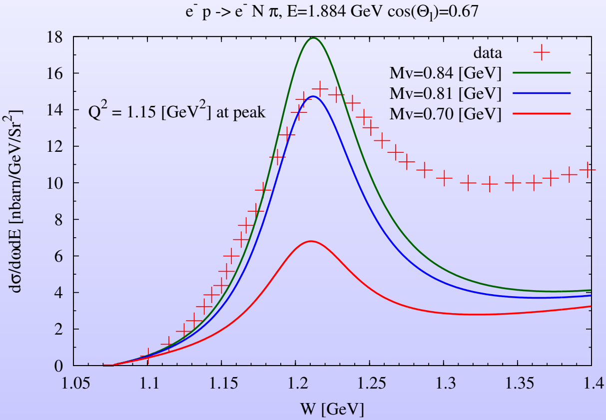
- Varying the vector mass:



- For low $Q^2 \approx 0.1[GeV^2]$ best $Mv_\Delta \approx 0.7[GeV]$

Vector Part Problem

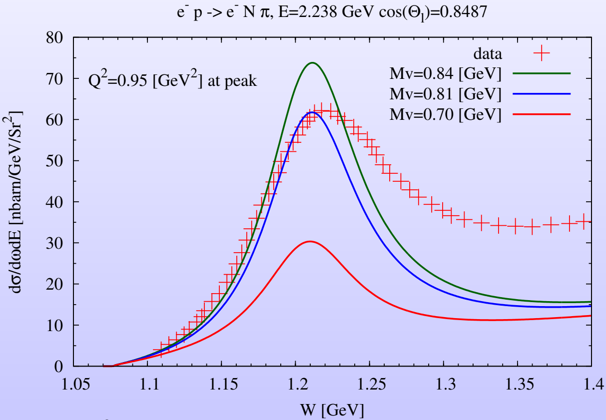
- Varying the vector mass:



- For $Q^2 \approx 1[GeV^2]$ best $Mv_\Delta \approx 0.81 - 0.82[GeV]$

Vector Part Problem

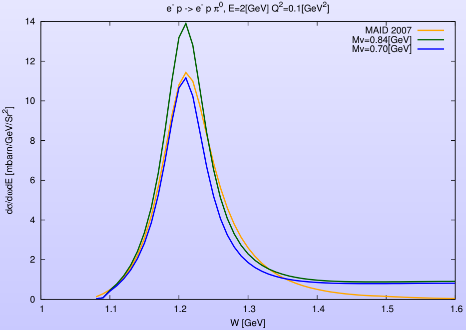
- Varying the vector mass:



- For $Q^2 \approx 1[GeV^2]$ best $Mv_\Delta \approx 0.81 - 0.82[GeV]$
- Approximate solution: $Mv(Q^2)$ parametrization

Vector Part Problem

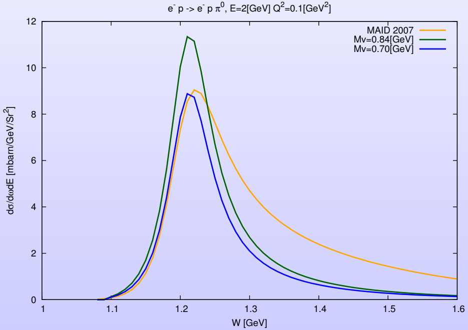
- Lack of good data for fixed Q^2, using MAID 2007
- Problem: different model in MAID 2007, background "*Born*" $+ \rho + \omega$.
- Δ+background comparison:



- Full model: varying M_v seems to work.

Vector Part Problem

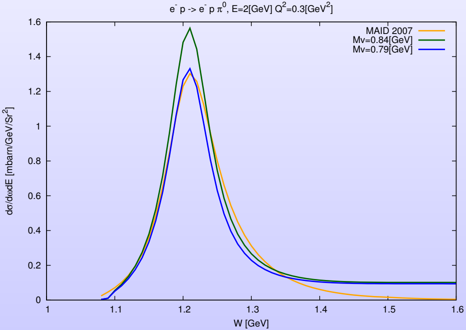
- The same for Δ only:



- OnlyΔ: totally different results even at this level.

Vector Part Problem

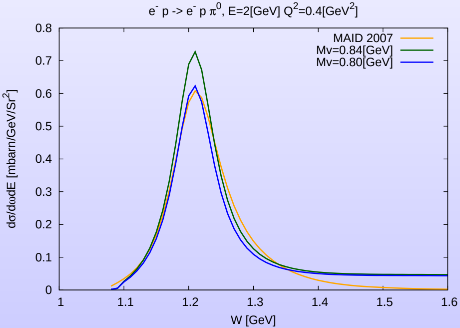
- A little higher Q^2



- Rapid growth of effective M_v to 0.79 [GeV]

Vector Part Problem

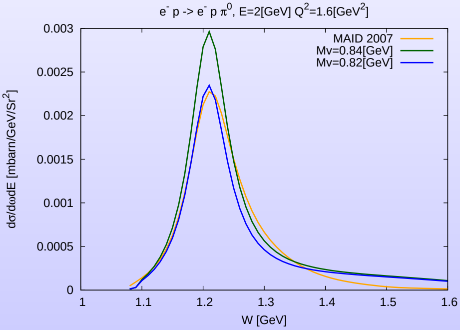
- A little higher Q^2



- Less rapid growth of effective M_v to 0.80 [GeV]

Vector Part Problem

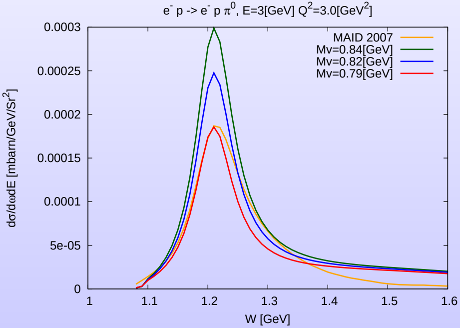
- Rather high Q^2



- For high Q^2 slow saturation of M_V?.

Vector Part Problem

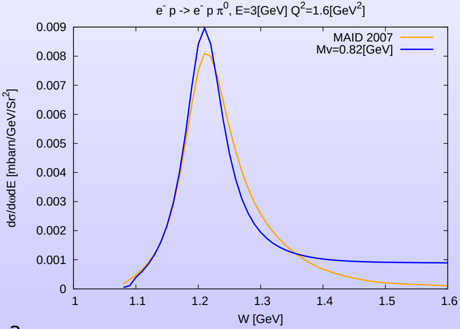
- Even higher Q^2 and E=3[GeV]:



- Not so good behaviour, M_v seems to drop!

Vector Part Problem

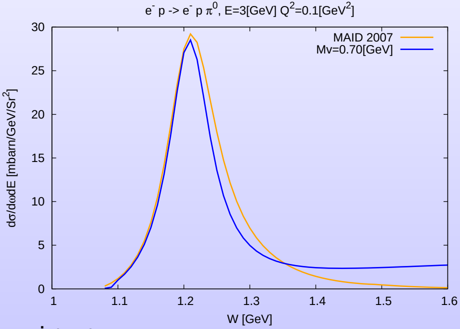
- Check of the results at $Q^2 = 1.6[GeV^2]$ for E=3[GeV]:



- Small difference?

Vector Part Problem

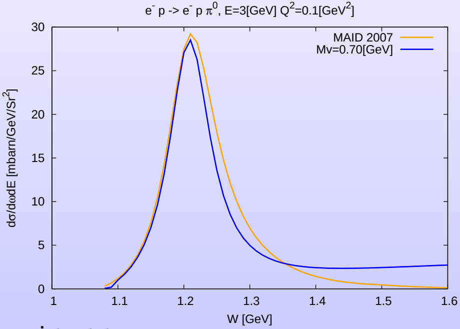
- Check of the results at $Q^2 = 0.1[GeV^2]$ for E=3[GeV]:



- Here it seems consistent

Vector Part Problem

- Check of the results at $Q^2 = 0.1[GeV^2]$ for E=1[GeV]:



- Here it seems consistent too.

Vector Part Problem

- Parametrization of the vector part uncertainty not trivial:
  - Approximate M_V solution is not monotonous.
  - At low $Q^2 = 0.1[GeV^2]$ the results for $E = 1[GeV]$ and $E = 3[GeV]$ consistent, at $Q^2 = 1.6[GeV^2]$ small neutrino energy dependence?
- Some estimation of the analysis bias possible.

Heavier resonances

- Around $W \approx 1.4[GeV]$: three additional isospin-1/2 resonances: $P_{11}^{\frac{1}{2},\frac{1}{2},+}(1440)$, $D_{13}^{\frac{3}{2},\frac{1}{2},-}(1520)$ and $S_{11}^{\frac{1}{2},\frac{1}{2},-}(1530)$. In neutrino scattring: only in the neutron channel.
- No separate amplitudes in the code, cross-sections for the resonance production used (T. Leitner and O. Buss dissertations):

$$\frac{d\sigma}{d\Omega' dE'} = \frac{|\mathbf{l'}|}{64\pi^2} \frac{A(p')}{\sqrt{(p \cdot l)^2 - m_l^2 M_N^2}} |M_R^2|$$

$$A(p') = \frac{\sqrt{p'^2}}{\pi} \frac{\Gamma_R(p'^2)}{(p'^2 - M_R^2)^2 + p'^2 \Gamma_R^2(p'^2)}$$

- Matrix element:

$$|M_R^2| = L_{\mu\nu} A_R^{\mu\nu}$$

Spin-1/2 Resonances

- For spin-1/2 resonances:

$$A_R^{\mu\nu} = \mathrm{Tr}\left[(\not{p} + M)\gamma^0\Gamma^{\mu\dagger}\gamma^0(\not{p}' + M')\Gamma^\nu\right]$$

- Here $M' = \sqrt{p'^2}$!

$$\begin{aligned} \Gamma^\mu &= (V^\mu - A^\mu)(\gamma^5) \\ V^\mu &= F_1(\gamma^\mu + \frac{q^\mu \not{q}}{Q^2})F_1 + i\sigma^{\mu\alpha}q_\alpha \frac{F_2}{2M} \\ -A^\mu &= G_A(Q^2)\left(\gamma^\mu + \frac{\not{q}\, q^\mu}{q^2 - m_\pi^2}\right)\gamma^5 \end{aligned}$$

- For the positive/negative parity resonances.
- Electromagnetic form-factors from MAID 2007 analysis. Axial: problematic, dipole approximation: $G_A(Q^2) = F_A(0)/(1 + Q^2/GeV^2)^2$.

Spin-3/2 Resonances

- For spin-3/2 resonances:

$$A_R^{\mu\nu} = \mathrm{Tr}\left[(\not{p}+M)\gamma^0\Gamma^{\mu\alpha\dagger}\gamma^0 P^{3/2}_{\alpha\beta}(p')\Gamma^{\nu\beta}\right]$$

$$P^{3/2}_{\alpha\beta}(p') = -(\not{p}'+M')\left(g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3}\frac{p'_\alpha p'_\beta}{M'^2} + \frac{p'_\alpha\gamma_\beta - p'_\beta\gamma_\alpha}{3M'}\right)$$

- Excitation vertex of D_{13} like for Δ, extra γ^5 for the negative parity.
- Electromagnetic form-factors from MAID 2007 analysis.
- Axial part: $C_5^A(0)$ from the Goldberger-Treiman relation, dipole ansatz. Only C_5^A and C_6^A nonzero, lack of precise data for the rest.

Resonance Decays

- Many decay channels with different dynamics and angular momenta: P_{11}: 69% πN (l=1), 22% $\pi\Delta$ (l=1), 9% σN (l=0) ; D_{13}: 59% πN (l=2), 5% $\pi\Delta$ (l=0), 15% $\pi\Delta$ (l=2), 21% ρN (l=0); S_{11}: 51% πN (l=0), 43% ηN (l=0), 3% ρN (l=0), 3% σN (l=1), 1% πP_{11} (l=0)
- 2-body decay widths: phenomenological formula (Manley-Salesky)

$$\Gamma_{R\to ab} = \Gamma^0_{R\to ab}\frac{\rho_{R\to ab}(W^2)}{\rho_{R\to ab}(M_R^2)}$$

$$\rho_{ab}(W^2) = \int dp_a^2 \int dp_b^2 A_a(p_a^2) A_b(p_b^2) \frac{p^{cm}_{R\to ab}}{W} B^2_{R\to ab}(p^{cm}_{R\to ab}, l) F^2_{ab}(W)$$

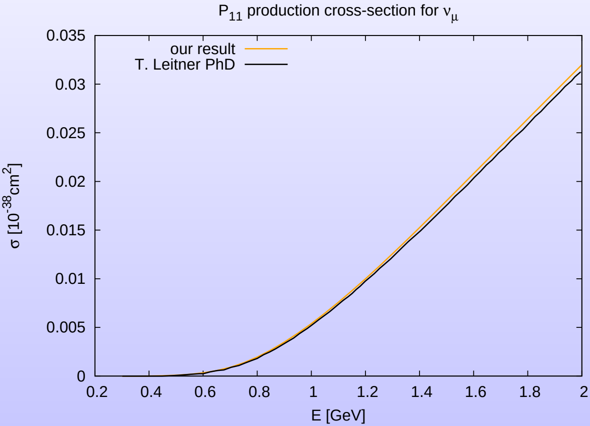
- Decay product spectral function $A(p^2)$: either $\delta(p^2-m^2)$ for stable particle or Breit-Wigner for unstable one, Blatt-Weisskopf centrifugal barrier $B(p^2, l)$ and a form-factor $F_{ab}(W)$.

2nd Resonance Region

- Unstable decay products: nested "widths", e.g. the $N\rho$ or $\pi\Delta$ channels.
- Simplification for the S_{11} : 51% πN, 49% ηN, η treated as stable. Rest :explicit.
- Theoretical problem: on the amplitude leve πN decay width from the relativistic decay vertex. Here: width from a different decay model!
- Still no solution for the background interference.

2nd Resonance Region, Comparison to T. Leitner

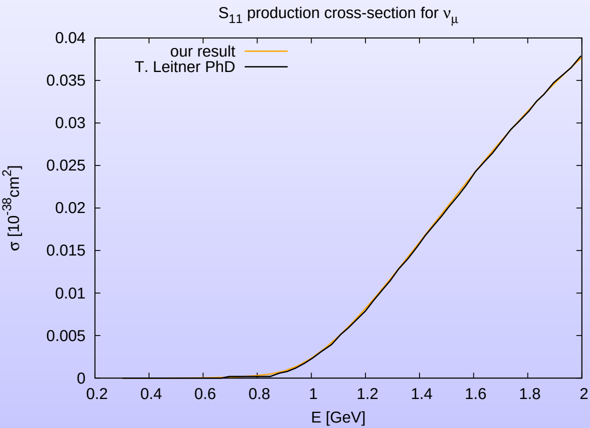
- Comarison of P_{11} total production cross-section:



- Good agreement.

2nd Resonance Region, Comparison to T. Leitner

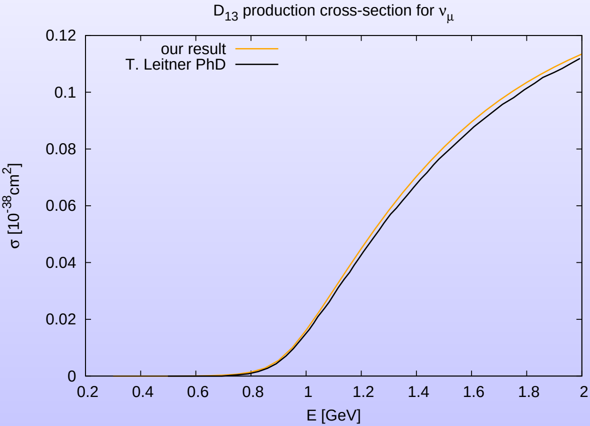
- Comarison of S_{11} total production cross-section:



- Good agreement.

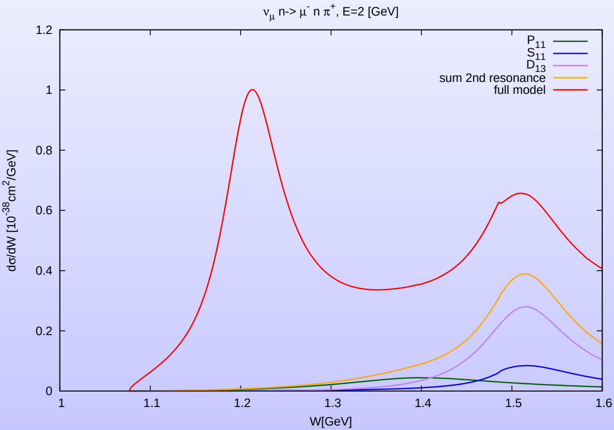
2nd Resonance Region, Comparison to T. Leitner

- Comarison of D_{13} total production cross-section:



- Good agreement.

2nd Resonance Region, Example differential cross-section

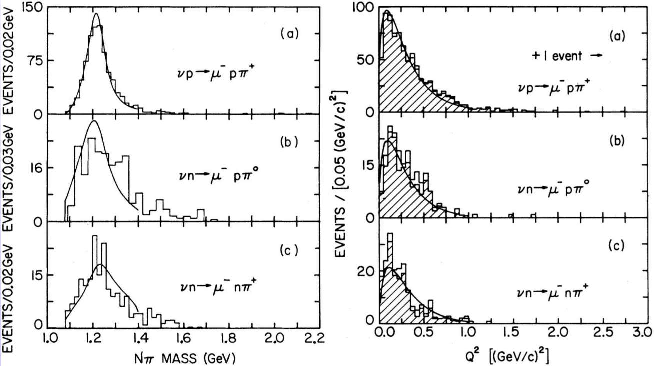


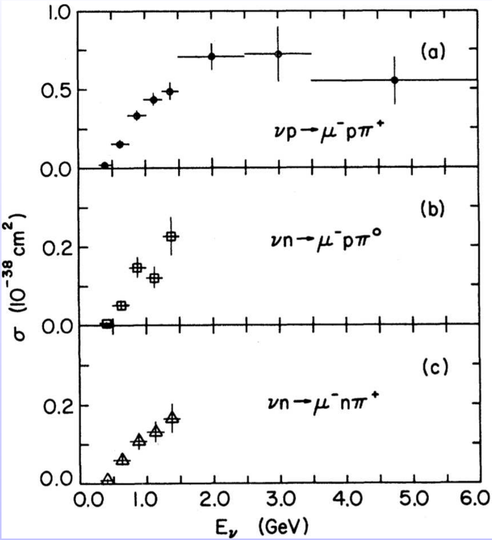
Second Resonance region may be important (depending on W cut and energy)

neutrino-deuteron scattering data

ANL data, Radecky82

- Averaged beam energy is around 0.7 GeV;
- Data collected for all three charged current SPP channels;
- Distributions of number of events depending on W, Q^2, E;
- dN/dQ^2 for $W < 1.4$ and $W_{no-limit}$ but from the dN/dW we see that $W_{no-limit} < 2$ GeV;
- For $\mu^-\pi^+ p$ $E \in (0.5, 6)$ GeV, in this case there are $d\sigma/dQ^2$ data;
- For $\mu^-\pi^0 p$ and $\mu^-\pi^+ n$, $E \in (0.3, 1.5)$ GeV;
- The total cross section data for all three channels;
- Some cuts for neutron channel data to avoid FSI.

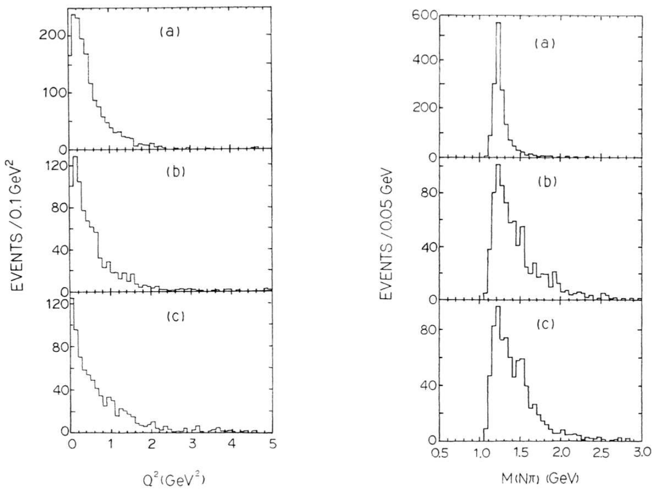

EVENTS/0.02GeV
EVENTS/0.03GeV
EVENTS/0.02GeV
(a)
(b)
(c)
νp → μ⁻ pπ⁺
νn → μ⁻ pπ°
νn → μ⁻ nπ⁺
Nπ MASS (GeV)
EVENTS / [0.05 (GeV/c)²]
+ 1 event →
Q² [(GeV/c)²]


(a)
νp → μ⁻pπ⁺
(b)
νn → μ⁻pπ⁰
(c)
νn → μ⁻nπ⁺
σ (10⁻³⁸ cm²)
1.0
0.5
0.0
0.2
0.0
0.2
0.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
E_ν (GeV)

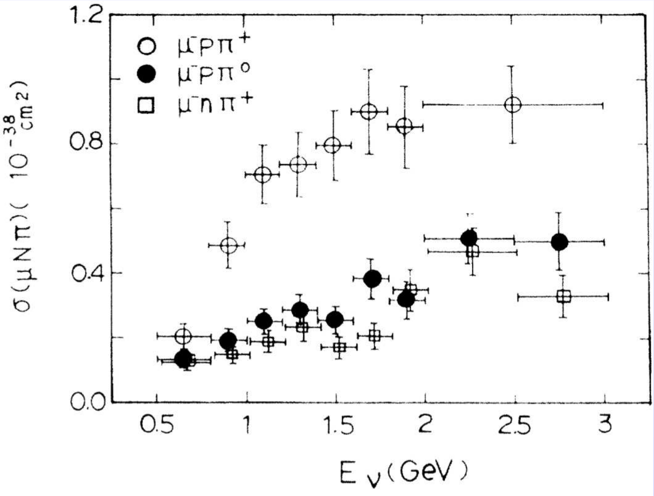
BNL data, Kitagaki86, Kitagaki90

- Averaged beam energy is around 1.3 GeV;
- Data collected for all three charged current SPP channels;
- Distributions of number of events depending on W, Q^2, E;
- dN/dQ^2 for $W_{no-limit} > 2$ but for $\mu^- p \pi^+$ exists dN/dQ^2 for $W < 1.4$ GeV;
- The total cross section data for all three channels $E \in (0.5, 3)$ GeV.

BNL data, Kitagaki86



BNL data, Kitagaki86



How to analyze the data

$$\chi^2 = \chi^2_{ANL}(\mu^- p\pi^+) + \chi^2_{ANL}(\mu^- n\pi^+) + \chi^2_{ANL}(\mu^- p\pi^0) + \chi^2_{BNL}, \tag{4}$$

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{N_{th,i} - N_i}{\Delta N_i} \right)^2 + \left(\frac{\frac{\sigma_{tot-th}}{\sigma_{tot-ex}} \cdot \frac{N_{exp}}{N^{th}} - 1}{r} \right)^2, \tag{5}$$

or equivalently by

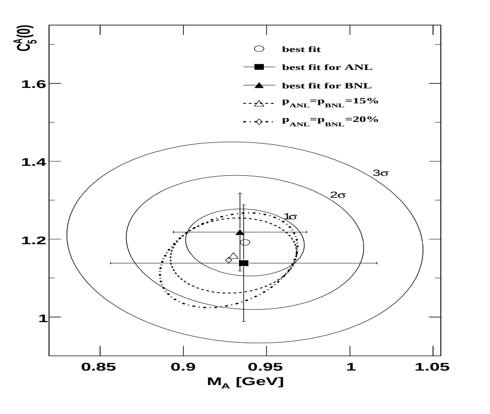
$$\chi^2 = \sum_{i=1}^{n} \left(\frac{\sigma^{diff}_{th}(Q_i^2) - p\sigma^{diff}_{ex}(Q_i^2)}{p\Delta\sigma_i} \right)^2 + \left(\frac{p-1}{r} \right)^2, \tag{6}$$

with:

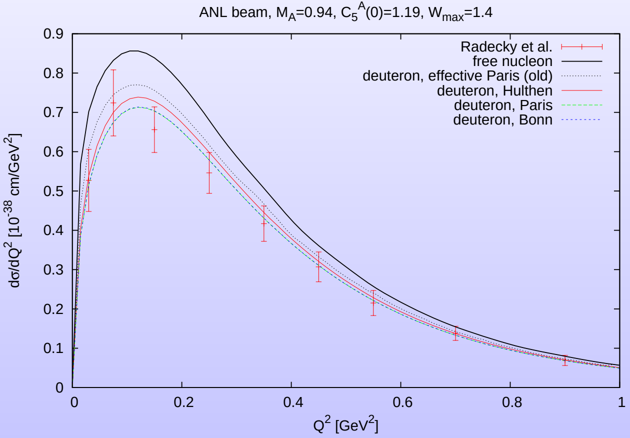
$$p \equiv \frac{\sigma_{tot-th}}{\sigma_{tot-exp}} \frac{N^{exp}}{N^{th}}, \tag{7}$$

but only one p_{ANL}

ANL and BNL data are consistent



deuteron correction



previous analysis results

	M_A (GeV)	$C_5^A(0)$	p_{ANL}	p_{BNL}	χ^2/NDF	GoF
only M_A, free target	0.95 ± 0.04		1.15 ± 0.06	0.98 ± 0.03	$25.5/28$	0.60
only M_A, deuteron	0.94 ± 0.04		1.04 ± 0.06	0.97 ± 0.03	$24.5/28$	0.65
M_A and $C_5^A(0)$, free target	$0.95 \pm 0.0 - 4$	1.14 ± 0.08	1.15 ± 0.11	0.98 ± 0.03	$25.5/27$	0.54
M_A and $C_5^A(0)$, deuteron	0.94 ± 0.03	1.19 ± 0.08	1.08 ± 0.10	0.98 ± 0.03	$24.3/27$	0.60

New results for νp scattering channel

- $M_A = 0.95 \pm 0.04$ GeV, $C_5^A(0) = 1.00 \pm 0.1$, with $M_V = 0.84$ GeV (full model with background), similarly as in Hernandez et al.
- $M_A = 1.17 \pm 0.04$ GeV $C_5^A(0) = 1.00 \pm 0.1$ with $M_V = 0.7$ GeV