Multipole expansion in single pion production

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D.Rein
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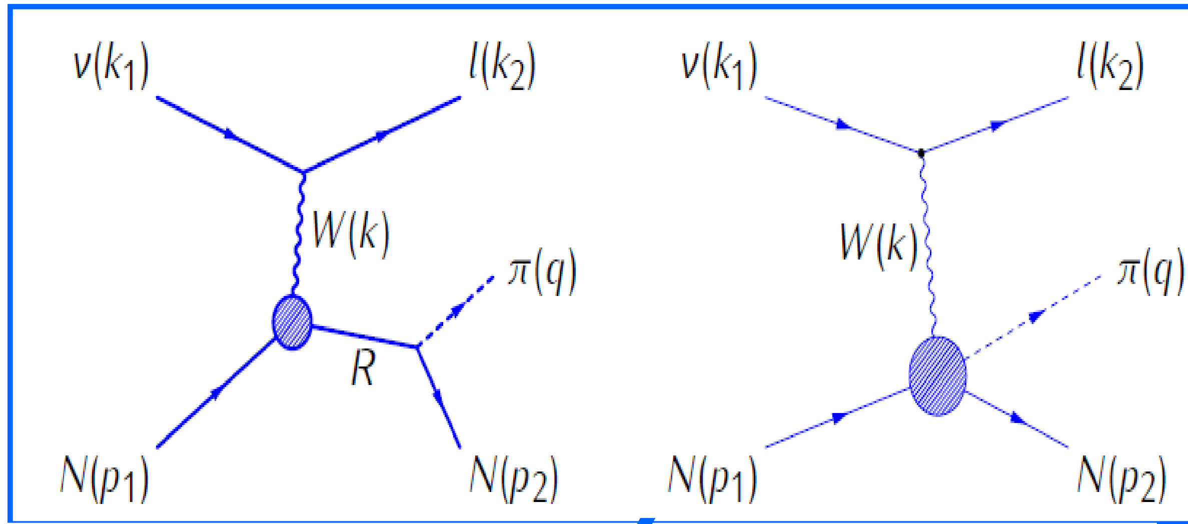
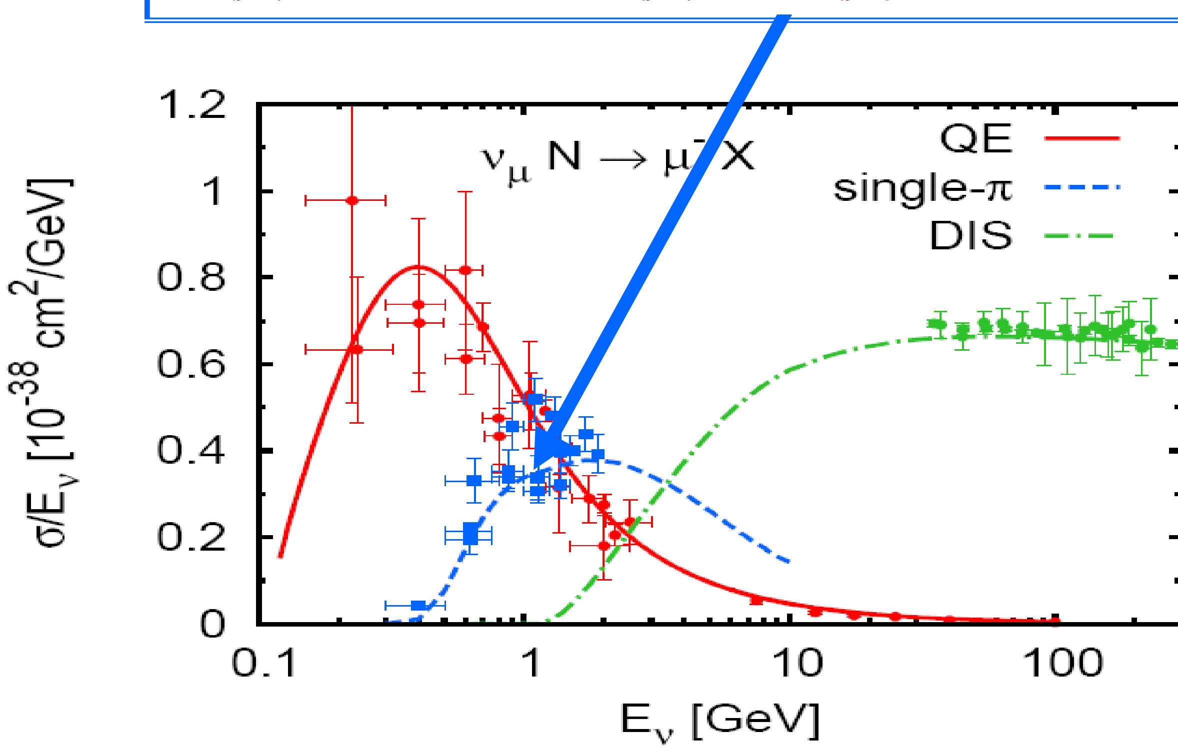
Single-Pion Production

$$\nu p \to \mu^- p \pi^+$$
$$\nu n \to \mu^- p \pi^0$$
$$\nu n \to \mu^- n \pi^+$$





In This model:

- Resonant interactions are described by Rein-Sehgal Model
- For non-resonant interactions 3 Born graphs are suggested
- Outgoing leptons are massless

Corrections:

- 2 more possible diagrams will be added to the model.
- non-zero lepton mass correction will be implemented.

Rein-Sehgal Model

- is based on **helicity amplitudes** derived in a relativistic quark model by Feynman, Kislingerand Ravndal (FKR).

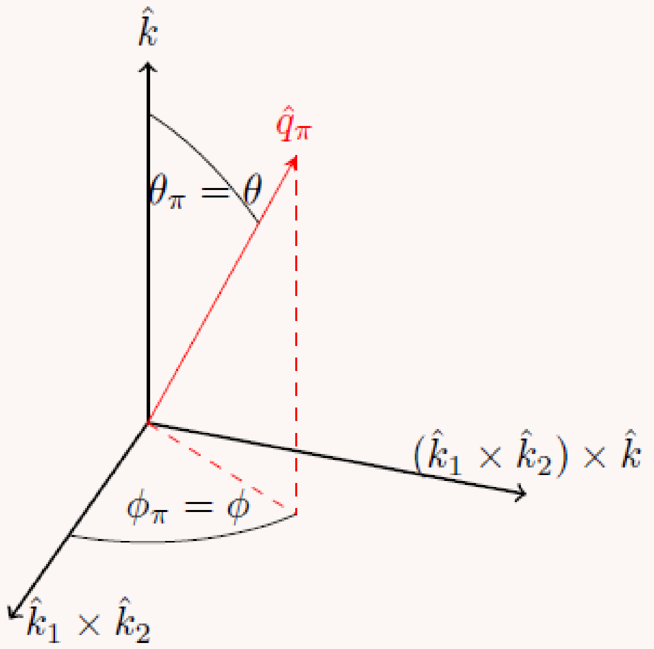
- The helicity amplitudes depend on the spin projection of the initial and final states, and on the transition currents F+ (F−, F0) corresponds to the gauge boson with positive (negative, zero) helicity:

$$
\begin{aligned}
f_{-3} &= \langle \mathcal{N}, \tfrac{1}{2} \,|\, F_- \,|\, \mathcal{N}^*, \tfrac{3}{2} \rangle, \\
f_{-1} &= \langle \mathcal{N}, -\tfrac{1}{2} \,|\, F_- \,|\, \mathcal{N}^*, \tfrac{1}{2} \rangle, \\
f_{+1} &= \langle \mathcal{N}, \tfrac{1}{2} \,|\, F_+ \,|\, \mathcal{N}^*, -\tfrac{1}{2} \rangle, \\
f_{+3} &= \langle \mathcal{N}, -\tfrac{1}{2} \,|\, F_+ \,|\, \mathcal{N}^*, -\tfrac{3}{2} \rangle, \\
f_{0\pm} &= \langle \mathcal{N}, \pm\tfrac{1}{2} \,|\, F_0 \,|\, \mathcal{N}^*, \pm\tfrac{1}{2} \rangle,
\end{aligned}
$$

$$
\frac{d\sigma^{CC}(\nu N \to l R \to l N \pi)}{dk^2\, dW} = \frac{G_F^2}{2} \cos^2\theta_c \frac{1}{(2\pi)^3}
\cdot \frac{(-k^2)}{(\mathbf{k}^L)^2} \cdot \frac{W^2}{M^2} \cdot \frac{\pi \Gamma_R x_E}{(W - M_R)^2 + \Gamma_R^2/4} |C_{N\pi}^I|^2
\cdot \{ u^2 (|f_{+1}^{CC}|^2 + |f_{+3}^{CC}|^2) + v^2 (|f_{-1}^{CC}|^2 + |f_{-3}^{CC}|^2)
+ 2uv \left(\frac{\mathbf{k}^2}{-k^2} \right) (|f_{0+}^{CC}|^2 + |f_{0-}^{CC}|^2) \}
$$

Plan for Non-resonant interactions

need to have a model to treat non-resonant contribution as resonant interaction in the same frame of description .

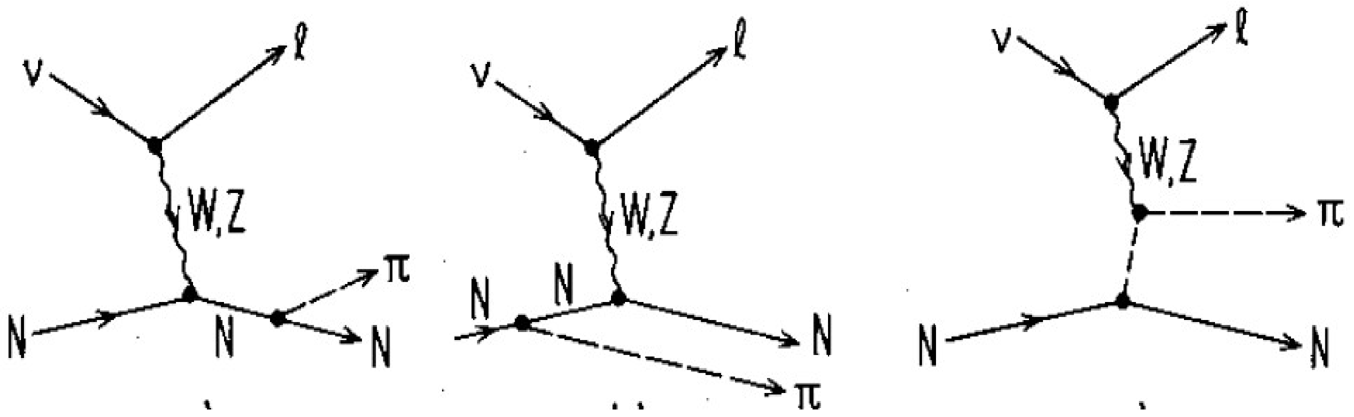


Coordinate frame in barycentric (πN center of mass) system

most suitable for discussing resonance contribution is a representation in terms of barycentric or isobar amplitudes.
We will chose this frame and try to calculate amplitudes of both interactions in this system.
z axis is along the momentum transfer (k)

Non-Resonant background

- A model for non-resonant background is provided by generalized **Born graphs** for single pion production



$$\mathcal{M}_{NN\pi} = \frac{G_F}{\sqrt{2}} \cos\theta_C g_{NN\pi} \frac{1}{s-M} \bar{u}(p_2)\gamma_5(\not{p}_1 + \not{k} + M)\epsilon^\mu \Gamma_\mu u(p_1)$$

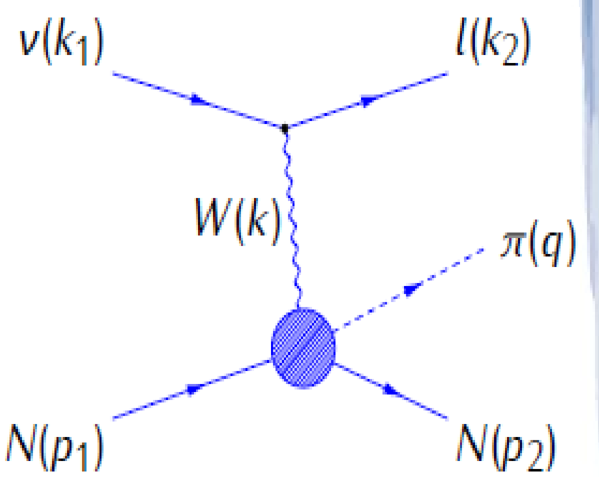
$$\mathcal{M}_{N} = \frac{G_F}{\sqrt{2}} \cos\theta_C g_{NN\pi} \frac{1}{u-M} \bar{u}(p_2)\epsilon^\mu \Gamma_\mu(\not{p}_2 - \not{k} + M)\gamma_5 u(p_1)$$

$$\mathcal{M}_{\pi} = \frac{G_F}{\sqrt{2}} \cos\theta_C g_{NN\pi} \frac{1}{t-m_\pi^2} F_\pi \gamma_5[2(\epsilon q) - (\epsilon k)]u(p_1)$$

$$\Gamma_\mu = -\{(F_1^V(k^2)\gamma_\mu - \frac{F_2^V(k^2)}{2M}[\gamma_\mu, \not{k}]) - F_A(k^2)\gamma_\mu\gamma_5\}$$

General Framework

$$\nu_l(k_1) + N(p_1) \to l(k_2) + N(p_2) + \pi(q)$$



$$\mathcal{M}^{(I)} = \frac{G_F}{\sqrt{2}} \cos\theta_C \; \epsilon^\rho \; \langle \; N\pi | \; J_\rho^{(I)} \; | N \; \rangle$$

$$= \frac{G_F}{\sqrt{2}} \cos\theta_C \; \epsilon^\rho \; a_{(I)}(V_\rho^{(I)} - A_\rho^{(I)})$$

$$\epsilon^\rho = \bar{u}_\mu(k_2)\gamma^\rho(1-\gamma_5)u_\nu(k_1)$$

Combination of amplitudes Corresponding to the outgoing hadrons Isospin

$$\mathcal{M}^{\frac{1}{2}} = 3\mathcal{M}_{NN\pi} - \mathcal{M}_N - 4\mathcal{M}_\pi$$

$$\mathcal{M}^{\frac{3}{2}} = 2\mathcal{M}_N + 2\mathcal{M}_\pi$$

$$\mathcal{M}^{\frac{1}{2}}_{Axial} = \frac{G_F}{\sqrt{2}} \cos\theta_C g_{NN\pi} F_A(k^2) \Big(\bar{u}(p_2) \frac{1}{s-M} \gamma_5 (\not{p}_1 + \not{k} + M) \epsilon^\mu \gamma_\mu \gamma_5 u(p_1)$$

$$- \bar{u}(p_2) \frac{1}{u-M^2} \epsilon^\mu \gamma_\mu \gamma_5 (\not{p}_2 - \not{k} + M) \gamma_5 u(p_1) \Big)$$

Isospin: Amplitude Decomposition

$$
\begin{aligned}
T_{NN\pi} &= \chi^\dagger \tau_a \phi_a^* \tau_+ W_+ \chi/2 = \chi^\dagger \phi_a^* (\frac{\tau_a \tau_+}{2}) W_+ \chi \\
T_N &= \chi^\dagger \tau_+ W_+ \tau_a \phi_a^* \chi/2 = \chi^\dagger W_+ (\frac{\tau_+ \tau_a}{2}) \phi_a^* \chi \\
T_\pi &= i\chi^\dagger [(\phi_\pi^* \times \phi_{\pi'}).W^+](\tau.\phi_{\pi'}^*)\chi = i\chi^\dagger (\epsilon_{+aa'} \phi_\pi^{*a} \phi_{\pi'}^{a'} W^+)(\tau_{a'} \phi_{\pi'}^{*a'})\chi \\
&= \chi^\dagger \phi_\pi^{*a} \phi_{\pi'}^{a'} W^+ (i\epsilon_{+aa'} \tau_{a'}) \phi_{\pi'}^{*a'})\chi = \chi^\dagger \phi_\pi^{*a} \phi_{\pi'}^{a'} W^+ [\frac{1}{2}(\tau_+ \tau_a - \tau_a \tau_+)] \phi_{\pi'}^{*a'})\chi
\end{aligned}
$$

$$
(\frac{\tau_a \tau_+}{2}) I_{NN\pi} + (\frac{\tau_a \tau_+}{2}) I_N + \frac{1}{2}(\tau_+ \tau_a - \tau_a \tau_+) I_\pi
$$

$$
= \frac{1}{2} I^{(3/2)} \tau_+ \tau_a + \tau_a \tau_+ (\frac{1}{3} I^{(1/2)} + \frac{1}{6} I^{(3/2)})
$$

$$
\begin{aligned}
I^{(1/2)} &= \frac{3}{2} I_{NN\pi} - \frac{1}{2} I_N - 2 I_\pi \\
I^{(3/2)} &= I_N + I_\pi
\end{aligned}
$$

Vector Current Conservation (CVC): $k_\mu J^\mu = 0$

$$
\begin{aligned}
J^\mu_{NN\pi} &= g_{NN\pi}\frac{1}{s-M}\bar{u}(p_2)\gamma_5(\not{p}_1+\not{k}+M)\Gamma^\mu u(p_1)\\
J^\mu_{N} &= g_{NN\pi}\frac{1}{u-M}\bar{u}(p_2)\Gamma^\mu(\not{p}_2-\not{k}+M)\gamma_5 u(p_1)\\
J^\mu_{\pi} &= g_{NN\pi}\frac{1}{t-m_\pi^2}F_\pi\gamma_5[2q^\mu-k^\mu]u(p_1)
\end{aligned}
$$

$$\Gamma_\mu = -\{(F_1^V(k^2)\gamma_\mu - \frac{F_2^V(k^2)}{2M}[\gamma_\mu,\not{k}])\}$$

$$
\begin{aligned}
\sigma_{\mu\nu}k_\mu k_\nu &= 0\\
k_\mu J^\mu_{NN\pi} &= -g_{NN\pi}\bar{u}(p_2)\gamma_5 F_1(k^2)u(p_1)\\
k_\mu J^\mu_{N} &= g_{NN\pi}\bar{u}(p_2)\gamma_5 F_1(k^2)u(p_1)\\
k_\mu J^\mu_{\pi} &= -g_{NN\pi}\bar{u}(p_2)\gamma_5 F_\pi(k^2)u(p_1)
\end{aligned}
$$

$$k_\mu J^\mu[I=\frac{1}{2}] = g_{NN\pi}\bar{u}(p_2)4\gamma_5[F_\pi(k^2)-F_1(k^2)]u(p_1)$$
$$k_\mu J^\mu[I=\frac{2}{3}] = g_{NN\pi}\bar{u}(p_2)2\gamma_5[F_1(k^2)-F_\pi(k^2)]u(p_1)$$

- $F_1(k^2) = F_\pi(k^2)$
- If we neglect lepton mass $k_\mu\epsilon^\mu = 0$

Terms proportional to $k_\mu\epsilon^\mu$ can be added to the transition amplitudes :

$$4g_{NN\pi}\gamma_5\frac{[F_1(k^2)-F_\pi(k^2)]}{k^2}k^\mu \rightarrow J^\mu[I=\frac{1}{2}]$$
$$-2g_{NN\pi}\gamma_5\frac{[F_1(k^2)-F_\pi(k^2)]}{k^2}k^\mu \rightarrow J^\mu[I=\frac{2}{3}]$$

1. Lorentz Covariance

Allows the vector and axial vector matrix elements to be decomposed:

$$\varepsilon^{\rho} V_{\rho}^{(I)} = \sum_{k=1}^{6} V_k^{(I)}(s,t,u)\cdot \bar{u}_N(p_2)\, O(V_k)\, u_N(p_1)$$

$$\varepsilon^{\rho} A_{\rho}^{(I)} = \sum_{k=1}^{8} A_k^{(I)}(s,t,u)\cdot \bar{u}_N(p_2)\, O(A_k)\, u_N(p_1)$$

Lorentz invariants $O(V_k)$ and $O(A_k)$ are linearly independent and can be used as a basis.

a) Vector invariants	b) Axial vector invariants
$O(V_1) = \frac{1}{2}\gamma_5\{\gamma, \gamma\}$	$O(A_1) = \frac{1}{2}[(\gamma q)(\gamma \varepsilon) - (\gamma \varepsilon)(\gamma q)]$
$O(V_2) = -2\gamma_5\{P, q\}$	$O(A_2) = 2(\varepsilon P)$
$O(V_3) = \gamma_5\{\gamma, q\}$	$O(A_3) = (\varepsilon q)$
$O(V_4) = 2\gamma_5\{\gamma, P\} - M\gamma_5\{\gamma, \gamma\}$	$O(A_4) = M(\varepsilon\gamma)$
$O(V_5) = -\gamma_5\{k, q\}$	$O(A_5) = -2(\gamma k)(\varepsilon P)$
$O(V_6) = \gamma_5\{k, \gamma\}$	$O(A_6) = -(\gamma k)(\varepsilon q)$
	$O(A_7) = (\varepsilon k)$
	$O(A_8) = -(\gamma k)(\varepsilon k)$

$$\{a, b\} \equiv (a\varepsilon)(bk) - (ak)(b\varepsilon)$$

$$\mathcal{M}^{(I)} = \frac{G}{\sqrt{2}} cos\theta_C \; \bar{u}(p_2) \; [\sum_{k=1}^{6} V_k^{(I)}(s,t,u) O(V_k) - \sum_{k=1}^{8} A_k^{(I)}(s,t,u) O(A_K)] \; u(P_1)$$

$$\mathcal{M}^{\frac{1}{2}}_{Axial} = \frac{G_F}{\sqrt{2}} \cos\theta_C g_{NN\pi} F_A(k^2) \Big(\bar{u}(p_2) \frac{1}{s-M} \gamma_5 (\not{p}_1 + \not{k} + M) \epsilon^\mu \gamma_\mu \gamma_5 u(p_1)$$

$$- \bar{u}(p_2) \frac{1}{u-M^2} \epsilon^\mu \gamma_\mu \gamma_5 (\not{p}_2 - \not{k} + M) \gamma_5 u(p_1) \Big)$$

$$\not{p} u(p) = M u(p)$$

$$-\epsilon^\mu A_\mu^{\frac{1}{2}} = g_{NN\pi} F_A(k^2) \Big(\bar{u}(p_2) (\frac{\not{q} \; \not{\epsilon}}{s-M} - \frac{\not{q} \; \not{\epsilon}}{u-M}) u(p_1) \Big)$$

$$\not{q} \; \not{\epsilon} = \frac{1}{2} [\not{q} \; \not{\epsilon} - \not{\epsilon} \; \not{q}] + q\epsilon = O(A_1) + O(A_3)$$

$$-\epsilon^\mu A_\mu^{\frac{1}{2}} = g_{NN\pi} F_A(k^2) \bar{u}(p_2) ((\frac{3}{s-M} + \frac{1}{u-M}) O(A_1) + (\frac{3}{s-M} - \frac{1}{u-M}) O(A_3)) u(p_1)$$

$$A_1^{(\frac{1}{2})} = -g_{NN\pi} F_A(k^2) (\frac{3}{s-M} + \frac{1}{u-M})$$

$$A_3^{(\frac{1}{2})} = -g_{NN\pi} F_A(k^2) (\frac{3}{s-M} - \frac{1}{u-M})$$

Table 10. Invariant Born-amplitudes $V_i^{(\rho)}$, $A_i^{(\rho)}$

Amplitude	$\rho=\frac{1}{2}$	$\rho=3/2$
$V_1^{(\rho)}$	$cF_1^V\left(\frac{3}{s-M^2}-\frac{1}{u-M^2}\right)$	$cF_1^V\frac{2}{u-M^2}$
$V_2^{(\rho)}$	$c\frac{F_1^V}{qk}\left(\frac{3}{s-M^2}-\frac{1}{u-M^2}\right)$	$c\frac{F_1^V}{qk}\cdot\frac{2}{u-M^2}$
$V_3^{(\rho)}$	$-c\frac{F_2^V}{M}\left(\frac{3}{s-M^2}+\frac{1}{u-M^2}\right)$	$c\frac{F_2^V}{M}\cdot\frac{2}{u-M^2}$
$V_4^{(\rho)}$	$-c\frac{F_2^V}{M}\left(\frac{3}{s-M^2}-\frac{1}{u-M^2}\right)$	$-c\frac{F_2^V}{M}\cdot\frac{2}{u-M^2}$
$V_5^{(\rho)}$	$-c\frac{4}{k^2}\left(\frac{F_1^V}{qk}+\frac{2F_\pi}{t-m_\pi^2}\right)$	$c\frac{2}{k^2}\left(\frac{F_1^V}{qk}+\frac{2F_\pi}{t-m_\pi^2}\right)$
$A_1^{(\rho)}$	$-cF_A\left(\frac{3}{s-M^2}+\frac{1}{u-M^2}\right)$	$+cF_A\frac{2}{u-M^2}$
$A_3^{(\rho)}$	$-cF_A\left(\frac{3}{s-M^2}-\frac{1}{u-M^2}\right)$	$-cF_A\frac{2}{u-M^2}$

2. Isobaric Expansion

$$q + p_2 = k + p_1 = 0$$

$$\varepsilon^\rho V_\rho^{(I)} = \sum_{k=1}^{6} \mathscr{F}_k^{(I)}(s,t,u) \cdot \chi_2^* \Sigma_k \chi_1$$

$$\varepsilon^\rho A_\rho^{(I)} = \sum_{k=1}^{8} \mathscr{G}_k^{(I)}(s,t,u) \cdot \chi_2^* \Lambda_k \chi_1$$

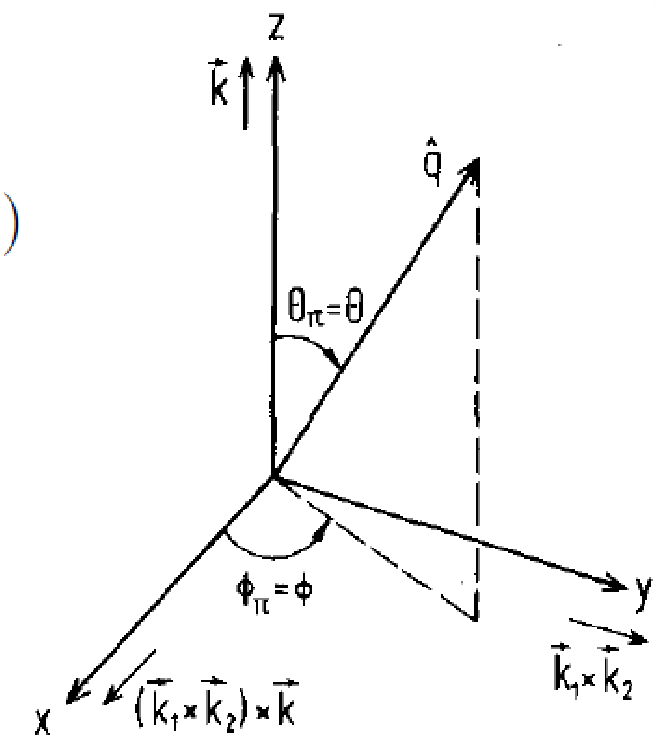
In the hadronic centre of frame the following choince of basis elements is made:

a) Vector	b) Axial vector
$\Sigma_1 = \boldsymbol{\sigma}\boldsymbol{\varepsilon} - \boldsymbol{\sigma}\hat{k}\cdot\hat{k}\boldsymbol{\varepsilon}$	$\Lambda_1 = -\boldsymbol{\sigma}\hat{q}\cdot(\boldsymbol{\sigma}\boldsymbol{\varepsilon} - \boldsymbol{\sigma}\hat{k}\cdot\hat{k}\boldsymbol{\varepsilon})$
$\Sigma_2 = -i\boldsymbol{\sigma}\hat{q}\cdot\boldsymbol{\sigma}(\hat{k}\times\boldsymbol{\varepsilon})$	$\Lambda_2 = i\boldsymbol{\sigma}(\hat{k}\times\boldsymbol{\varepsilon})$
$\Sigma_3 = \boldsymbol{\sigma}\hat{k}\cdot(\hat{q}\boldsymbol{\varepsilon} - \hat{q}\hat{k}\cdot\hat{k}\boldsymbol{\varepsilon})$	$\Lambda_3 = -\boldsymbol{\sigma}\hat{q}\cdot\boldsymbol{\sigma}\hat{k}(\hat{q}\boldsymbol{\varepsilon} - \hat{q}\hat{k}\cdot\hat{k}\boldsymbol{\varepsilon})$
$\Sigma_4 = \boldsymbol{\sigma}\hat{q}\cdot(\hat{q}\boldsymbol{\varepsilon} - \hat{q}\hat{k}\cdot\hat{k}\boldsymbol{\varepsilon})$	$\Lambda_4 = -(\hat{q}\boldsymbol{\varepsilon} - \hat{q}\hat{k}\cdot\hat{k}\boldsymbol{\varepsilon})$
$\Sigma_5 = \boldsymbol{\sigma}\hat{k}\cdot(k_0\hat{k}\boldsymbol{\varepsilon} - \varepsilon_0\lvert\mathbf{k}\rvert)$	$\Lambda_5 = -\boldsymbol{\sigma}\hat{q}\cdot\boldsymbol{\sigma}\hat{k}(k_0\cdot\hat{k}\boldsymbol{\varepsilon} - \varepsilon_0\lvert\mathbf{k}\rvert)/k^2$
$\Sigma_6 = \boldsymbol{\sigma}\hat{q}\cdot(k_0\cdot\hat{k}\boldsymbol{\varepsilon} - \varepsilon_0\lvert\mathbf{k}\rvert)$	$\Lambda_6 = -(k_0\cdot\hat{k}\boldsymbol{\varepsilon} - \varepsilon_0\lvert\mathbf{k}\rvert)/k^2$
	$\Lambda_7 = -\boldsymbol{\sigma}\hat{q}\cdot\boldsymbol{\sigma}\hat{k}(k\varepsilon)/k_0$
	$\Lambda_8 = -(k\varepsilon)/k_0$

Helicity Amplitudes

$$\tilde{F}^{(\lambda_k)}_{\lambda_2,\lambda_1} = \sum_{k=1}^{6} \mathscr{F}_k(s,t,u)\ \chi_2^*(\lambda_2)\ \Sigma_k(\lambda_k)\ \chi_1(\lambda_1)$$

$$\tilde{G}^{(\lambda_k)}_{\lambda_2,\lambda_1} = \sum_{k=1}^{8} \mathscr{G}_k(s,t,u)\ \chi_2^*(\lambda_2)\ \Lambda_k(\lambda_k)\ \chi_1(\lambda_1)$$



$$\mathbf{k} = |\mathbf{k}|(0,0,1)$$

$$\mathbf{q} = |\mathbf{q}|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

Gauge boson's polarization vectors

$$\varepsilon^{(\pm)} = \mp\frac{1}{\sqrt{2}}(1, \pm i, 0), \qquad \varepsilon_0^{(\pm)} = 0$$

$$\varepsilon^{(s)} = \frac{1}{\sqrt{-k^2}}(0, 0, k_0), \qquad \varepsilon_0^{(s)} = \frac{1}{\sqrt{-k^2}}|\mathbf{k}|$$

Nucleon spinors

$$\chi_1(\uparrow) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \chi_1(\downarrow) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\chi_2(\uparrow) = \begin{pmatrix} \sin\theta/2 \\ -e^{i\phi}\cos\theta/2 \end{pmatrix}, \qquad \chi_2(\downarrow) = \begin{pmatrix} e^{i\phi}\cos\theta/2 \\ \sin\theta/2 \end{pmatrix}$$

$\boldsymbol{\varepsilon}$	λ_k	λ_2	λ_1	$\tilde{F}^{(\lambda_k)}_{\lambda_2\lambda_1}$
$\boldsymbol{\varepsilon}^{(+)}$	$+1$	$\frac{1}{2}$	$\frac{1}{2}$	$-\sqrt{2}\left[\sin\frac{\theta}{2}(\mathscr{F}_1+\mathscr{F}_2)+\frac{1}{2}\sin\theta\cos\frac{\theta}{2}(\mathscr{F}_3+\mathscr{F}_4)\right]$
		$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}e^{i\phi}\sin\theta\sin\frac{\theta}{2}(\mathscr{F}_3-\mathscr{F}_4)$
		$-\frac{1}{2}$	$\frac{1}{2}$	$-\sqrt{2}\,e^{i\phi}\left[\cos\frac{\theta}{2}(\mathscr{F}_1-\mathscr{F}_2)-\frac{1}{2}\sin\theta\sin\frac{\theta}{2}(\mathscr{F}_3-\mathscr{F}_4)\right]$
		$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}e^{2i\phi}\sin\theta\cos\frac{\theta}{2}(\mathscr{F}_3+\mathscr{F}_4)$
$\boldsymbol{\varepsilon}^{(-)}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}e^{-2i\phi}\sin\theta\cos\frac{\theta}{2}(\mathscr{F}_3+\mathscr{F}_4)$
		$\frac{1}{2}$	$-\frac{1}{2}$	$\sqrt{2}\,e^{-i\phi}\left[\cos\frac{\theta}{2}(\mathscr{F}_1-\mathscr{F}_2)-\frac{1}{2}\sin\theta\sin\frac{\theta}{2}(\mathscr{F}_3-\mathscr{F}_4)\right]$
		$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{\sqrt{2}}e^{-i\phi}\sin\theta\sin\frac{\theta}{2}(\mathscr{F}_3-\mathscr{F}_4)$
		$-\frac{1}{2}$	$-\frac{1}{2}$	$-\sqrt{2}\left[\sin\frac{\theta}{2}(\mathscr{F}_1+\mathscr{F}_2)+\frac{1}{2}\sin\theta\cos\frac{\theta}{2}(\mathscr{F}_3+\mathscr{F}_4)\right]$
$\boldsymbol{\varepsilon}^{(s)}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\sqrt{-k^2}\,e^{-i\phi}\cos\frac{\theta}{2}(\mathscr{F}_5+\mathscr{F}_6)$
		$\frac{1}{2}$	$-\frac{1}{2}$	$\sqrt{-k^2}\sin\frac{\theta}{2}(\mathscr{F}_5-\mathscr{F}_6)$
		$-\frac{1}{2}$	$\frac{1}{2}$	$\sqrt{k^2}\sin\frac{\theta}{2}(\mathscr{F}_5-\mathscr{F}_6)$
		$-\frac{1}{2}$	$-\frac{1}{2}$	$\sqrt{-k^2}\,e^{i\phi}\cos\frac{\theta}{2}(\mathscr{F}_5+\mathscr{F}_6)$

3. equating two expansions

- **Lorentz Covariance**

$$\epsilon^{\rho} V_{\rho}^{(I)} = \sum_{k=1}^{6} V_k^{(I)}(s,t,u)\ \bar{u}_N(p_2) O(V_k) u_N(p_1)$$

$$\epsilon^{\rho} A_{\rho}^{(I)} = \sum_{k=1}^{8} A_k^{(I)}(s,t,u)\ \bar{u}_N(p_2) O(A_k) u_N(p_1)$$

- **Isobaric Expansion**

$$\epsilon^{\rho} V_{\rho}^{(I)} = \sum_{k=1}^{6} \mathscr{F}_k^{(I)}(s,t,u)\ \chi_2^* \ \Sigma_k\ \chi_1$$

$$\epsilon^{\rho} A_{\rho}^{(I)} = \sum_{k=1}^{8} \mathscr{G}_k^{(I)}(s,t,u)\ \chi_2^* \ \Lambda_k\ \chi_1$$

Two expansions can be equated by using:

$$u(p_i) = \sqrt{\frac{E_i+M}{2M}} \begin{pmatrix} \chi_i \\ \frac{\sigma . p_i}{E_i+M}\chi_i \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}$$

$$\mathcal{F}_i = K_i^V \cdot F_i / 2M \; (i = 1, \ldots, 6)$$

$$K_1^V = W_- O_{1+} \qquad K_4^V = q^2 W_- O_{2+}$$
$$K_2^V = W_+ O_{1-} \qquad K_5^V = 1/O_{2+}$$
$$K_3^V = q^2 W_+ O_{2-} \qquad K_6^V = 1/O_{2-}$$

$$F_1 = V_1 + qk(V_3 - V_4)/W_- + W_- V_4$$
$$F_2 = -V_1 + qk(V_3 - V_4)/W_+ + W_+ W_4$$
$$F_3 = V_3 - V_4 + V_{25}/W_+$$
$$F_4 = V_3 - V_4 - V_{25}/W_-$$
$$F_5 = (W_+^2 - k^2) V_1/2W - qk(W_+^2 - k^2 + 2WW_-) V_2/2W + (W_+ q_0 - qk)(V_3 - V_4) + (W_+^2 - k^2) W_- V_4/2W - k_0 qk V_5 + q_0 V_{25}$$
$$F_6 = -(W_-^2 - k^2) V_1/2W + qk(W_+^2 - k^2 + 2WW_-) V_2/2W + (W_- q_0 - qk)(V_3 - V_4) + (W_-^2 - k^2) W_+ V_4/2W + k_0 qk V_5 - q_0 V_{25}$$

$$W_\pm = W \pm M$$
$$O_{1\pm} = \{(W_\pm^2 - k^2)(W_\pm^2 - m_\pi^2)\}^{\frac{1}{2}}/2W$$
$$O_{2\pm} = \{(W_\pm^2 - k^2)/(W_\pm^2 - m_\pi^2)\}^{\frac{1}{2}}$$

$$V_{25} = W_+ W_- V_2 + k^2 V_5$$

Multipole Expansion

$$
\begin{aligned}
G_{\mu\nu}(\theta,\phi) &= \sum_j G^j_{\mu\nu}(2j+1)d^j_{\mu\nu}(\theta)e^{i(\lambda-\mu)\phi} \\
F_{\mu\nu}(\theta,\phi) &= \sum_j F^j_{\mu\nu}(2j+1)d^j_{\mu\nu}(\theta)e^{i(\lambda-\mu)\phi}
\end{aligned}
$$

$$
\lambda = \lambda_k - \lambda_1
$$

$$
\mu = \lambda_q - \lambda_2 = -\lambda_2
$$

The expansion coefficients refer to final pion_nucleon states of definite total angular momentum, but not definite parity. To have parity eigenstates we sum or subtract amplitudes with opposite helicity quantum number. Like:

$$
A^A{}_{\substack{l+\\ l+1-}} = \mp\frac{1}{\sqrt{2}}\left(G^j_{\frac12\frac12} \pm G^j_{-\frac12\frac12}\right) \quad , \qquad A^V{}_{\substack{l+\\ l+1-}} = \mp\frac{1}{\sqrt{2}}\left(F^j_{\frac12\frac12} \pm F^j_{-\frac12\frac12}\right)
$$

$$
B^A{}_{\substack{l+\\ l+1-}} = \pm\sqrt{\frac{2}{l(l+2)}}\left(G^j_{\frac12\frac32} \pm G^j_{-\frac12\frac32}\right) \quad , \qquad B^V{}_{\substack{l+\\ l+1-}} = \pm\sqrt{\frac{2}{l(l+2)}}\left(F^j_{\frac12\frac32} \pm F^j_{-\frac12\frac32}\right)
$$

$$
S^A{}_{\substack{l+\\ l+1-}} = \frac{\sqrt{-k^2}}{|\mathbf{k}|}\,\frac{1}{\sqrt{2}}\left(G^{0j}_{\frac12\frac12} \pm G^{0j}_{-\frac12\frac12}\right) \quad , \qquad S^V{}_{\substack{l+\\ l+1-}} = \frac{\sqrt{-k^2}}{|\mathbf{k}|}\,\frac{1}{\sqrt{2}}\left(F^{0j}_{\frac12\frac12} \pm F^{0j}_{-\frac12\frac12}\right)
$$

Cross-Section

$$\frac{d\sigma}{dk^2 dW} = [\frac{d\sigma}{dk^2 dW}]^0 + [\frac{d\sigma}{dk^2 dW}]^1 + [\frac{d\sigma}{dk^2 dW}]^2 + \ldots$$

$$\frac{d\sigma(\nu N \to l N\pi)}{dk^2 dW} = \frac{G_F^2}{2}\,\frac{1}{(2\pi)^3}\;|\mathbf{q}|\frac{(-k^2)}{(\mathbf{k^L})^2}|a_I(N\pi)|^2$$

$$\left\{u^2 \sum_{l=0}^{\infty}(l+1)\left[|A^V_{l+} + A^A_{l+}|^2 + |A^V_{l+1-} + A^A_{l+1-}|^2 + \frac{l(l+2)}{4}(|B^V_{l+} + B^A_{l+}|^2 + |B^A_{l+1-} + B^A_{l+1-}|^2)\right]\right.$$

$$+\; \left\{v^2 \sum_{l=0}^{\infty}(l+1)\left[|A^V_{l+} - A^A_{l+}|^2 + |A^V_{l+1-} - A^A_{l+1-}|^2 + \frac{l(l+2)}{4}(|B^V_{l+} - B^A_{l+}|^2 + |B^A_{l+1-} - B^A_{l+1-}|^2)\right]\right.$$

$$-\; 2uv(\frac{\mathbf{k^2}}{-k^2})\sum_{l=0}^{\infty}(l+1)\left[|S^V_{l+} + S^A_{l+}|^2 + |S^V_{l+1-} + S^A_{l+1-}|^2 + |S^V_{l+} - S^A_{l+}|^2 + |S^V_{l+1-} - S^A_{l+1-}|^2\right]\;\right\}$$

Resonance Contributions

$$\frac{d\sigma^{CC}(\nu N \to lR \to lN\pi)}{dk^2\,dW} = \frac{G_F^2}{2}\cos^2\theta_c \frac{1}{(2\pi)^3}$$
$$\cdot \frac{(-k^2)}{(\mathbf{k}^L)^2}\cdot\frac{W^2}{M^2}\cdot\frac{\pi\Gamma_R x_E}{(W-M_R)^2+\Gamma_R^2/4}|C_{N\pi}^I|^2$$
$$\cdot\left\{u^2(|f_{+1}^{CC}|^2+|f_{+3}^{CC}|^2)+v^2(|f_{-1}^{CC}|^2+|f_{-3}^{CC}|^2)\right.$$
$$\left.+2uv\left(\frac{\mathbf{k}^2}{-k^2}\right)(|f_{0+}^{CC}|^2+|f_{0-}^{CC}|^2)\right\}$$

$$f_{-3} = \langle \mathcal{N}, \tfrac{1}{2} | F_- | \mathcal{N}^*, \tfrac{3}{2}\rangle,$$
$$f_{-1} = \langle \mathcal{N}, -\tfrac{1}{2} | F_- | \mathcal{N}^*, \tfrac{1}{2}\rangle,$$
$$f_{+1} = \langle \mathcal{N}, \tfrac{1}{2} | F_+ | \mathcal{N}^*, -\tfrac{1}{2}\rangle,$$
$$f_{+3} = \langle \mathcal{N}, -\tfrac{1}{2} | F_+ | \mathcal{N}^*, -\tfrac{3}{2}\rangle,$$
$$f_{0\pm} = \langle \mathcal{N}, \pm\tfrac{1}{2} | F_0 | \mathcal{N}^*, \pm\tfrac{1}{2}\rangle,$$

$$\frac{d\sigma^{CC}(\nu N \to lR \to lN\pi)}{dk^2\,dW} = \frac{G_F^2}{2}\cos^2\theta_c \frac{1}{(2\pi)^3}$$
$$\cdot \frac{(-k^2)}{(\mathbf{k}^L)^2}\cdot|\mathbf{q}|\cdot\frac{2j+1}{2}|a_{CC}^I(N\pi)|^2$$
$$\cdot\left\{u^2\left(|A_{l+}^{(I)V}+A_{l+}^{(I)A}|^2+\frac{l(l+2)}{4}|B_{l+}^{(I)V}+B_{l+}^{(I)A}|^2\right)\right.$$
$$+v^2\left(|A_{l+}^{(I)V}-A_{l+}^{(I)A}|^2+\frac{l(l+2)}{4}|B_{l+}^{(I)V}-B_{l+}^{(I)A}|^2\right)$$
$$\left.+2uv\left(\frac{\mathbf{k}^2}{-k^2}\right)|S_{l+}^{(I)V}+S_{l+}^{(I)A}|^2+|S_{l+}^{(I)V}-S_{l+}^{(I)A}|^2\right)\right\}$$

$$A_{l\pm}^{(I)V} + A_{l\pm}^{(I)A} = \pm\sigma^D(R)\, C_{N\pi}^I\, \hat{\kappa}\, f_{\mathrm{BW}}(R) \cdot f_{+1}^{CC}(R(I, j = l \pm \tfrac{1}{2}))$$

$$A_{l\pm}^{(I)V} - A_{l\pm}^{(I)A} = -\sigma^D(R)\, C_{N\pi}^I\, \hat{\kappa}\, f_{BW}(R) \cdot f_{-1}^{CC}(R(I, j = l \pm \tfrac{1}{2}))$$

$$\sqrt{\frac{l(l+2}{4}}\,(B_{l\pm}^{(I)V} + B_{l\pm}^{(I)A}) = \mp\sigma^D(R)\, C_{N\pi}^I\, \hat{\kappa}\, f_{BW}(R) \cdot f_{+3}^{CC}(R(I, j = l \pm \tfrac{1}{2}))$$

$$\sqrt{\frac{l(l+2)}{4}}\,(B_{l\pm}^{(I)V} - B_{l\pm}^{(I)A}) = +\sigma^D(R)\, C_{N\pi}^I\, \hat{\kappa}\, f_{\mathrm{BW}}(R) \cdot f_{-3}^{CC}(R(I, j = l \pm \tfrac{1}{2}))$$

$$S_{l\pm}^{(I)V} + S_{l\pm}^{(I)A} = +\sigma^D(R)\, C_{N\pi}^I\, \hat{\kappa}\, f_{\mathrm{BW}}(R) \cdot f_{0-}^{CC}(R(I, j = l \pm \tfrac{1}{2}))$$

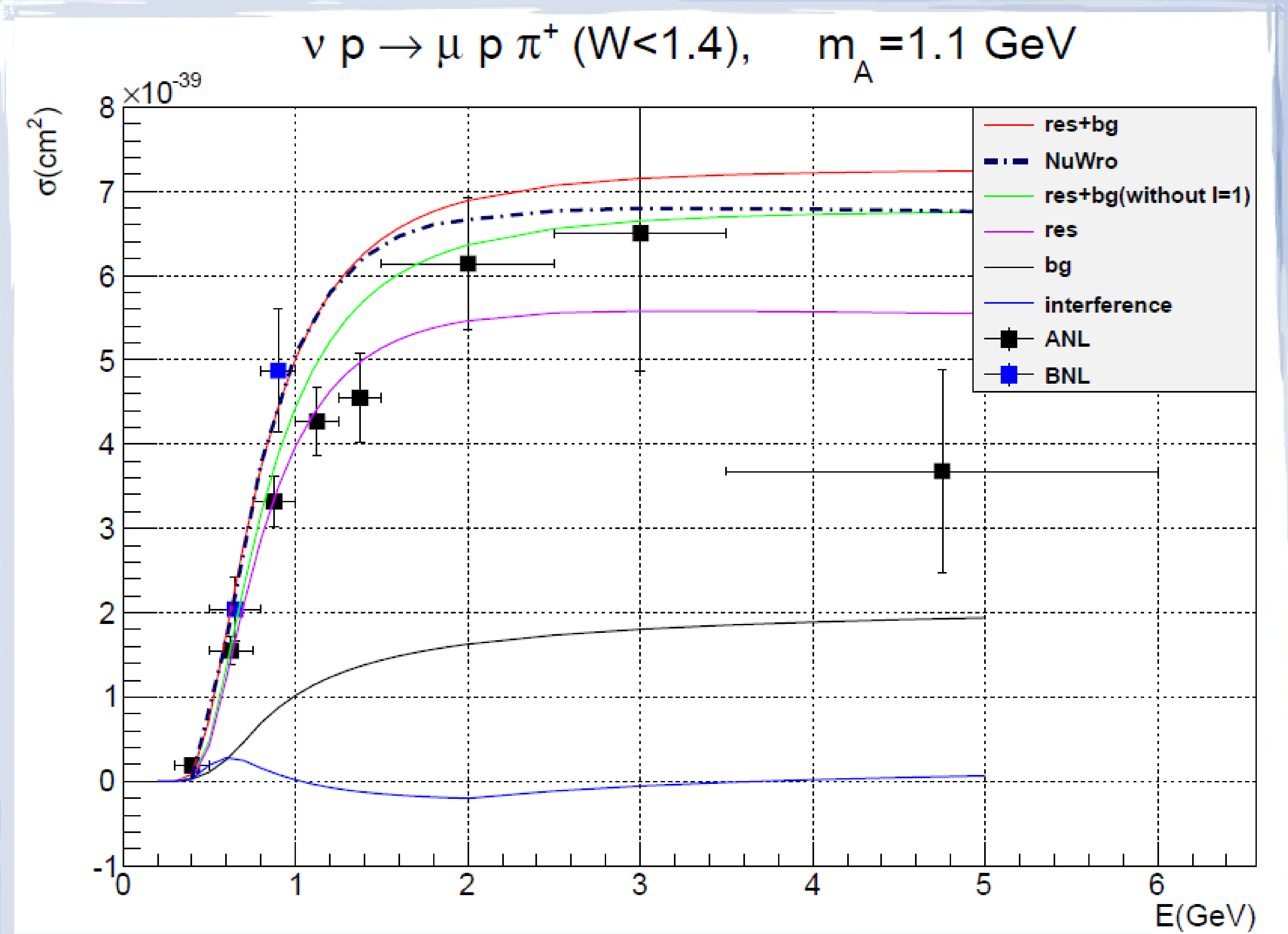
$$S_{l\pm}^{(I)V} - S_{l\pm}^{(I)A} = \mp\sigma^D(R)\, C_{N\pi}^I\, \hat{\kappa}\, f_{\mathrm{BW}}(R) \cdot f_{0+}^{CC}(R(I, j = l \pm \tfrac{1}{2}))$$

$$f_{\mathrm{BW}}(R) = \sqrt{\frac{\Gamma_R\, x_E}{2\pi}} \cdot \left(\frac{-1}{W - M_R + i\Gamma_R/2}\right)$$

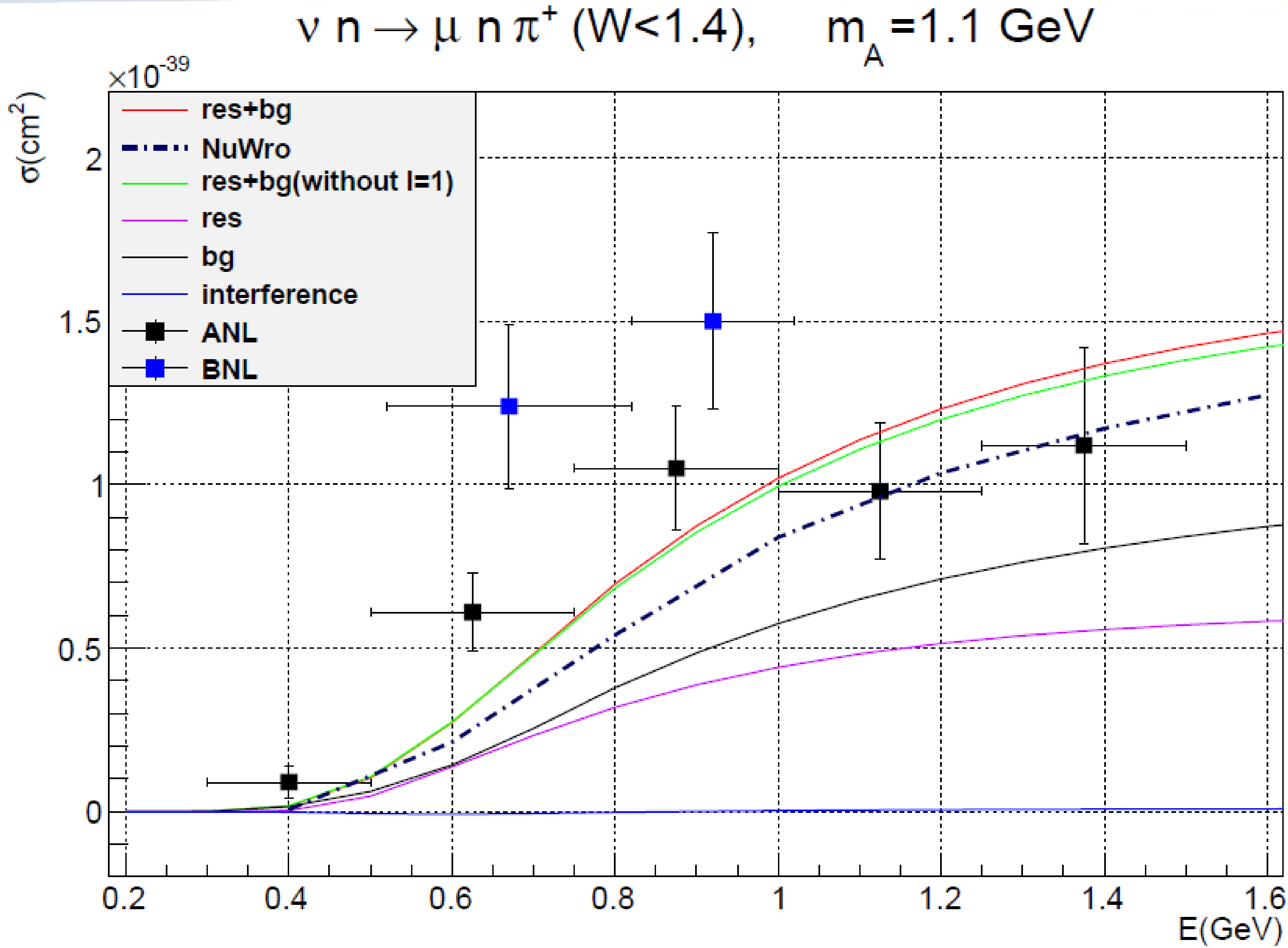
$$k \;=\; 2\pi\, \frac{W}{M}\, \frac{1}{\sqrt{2}j+1}\, \frac{1}{\sqrt{|q|}}\, \frac{C^{(I)}}{|C^{(I)}|a^{(I)}}$$

Lepton mass Implementation

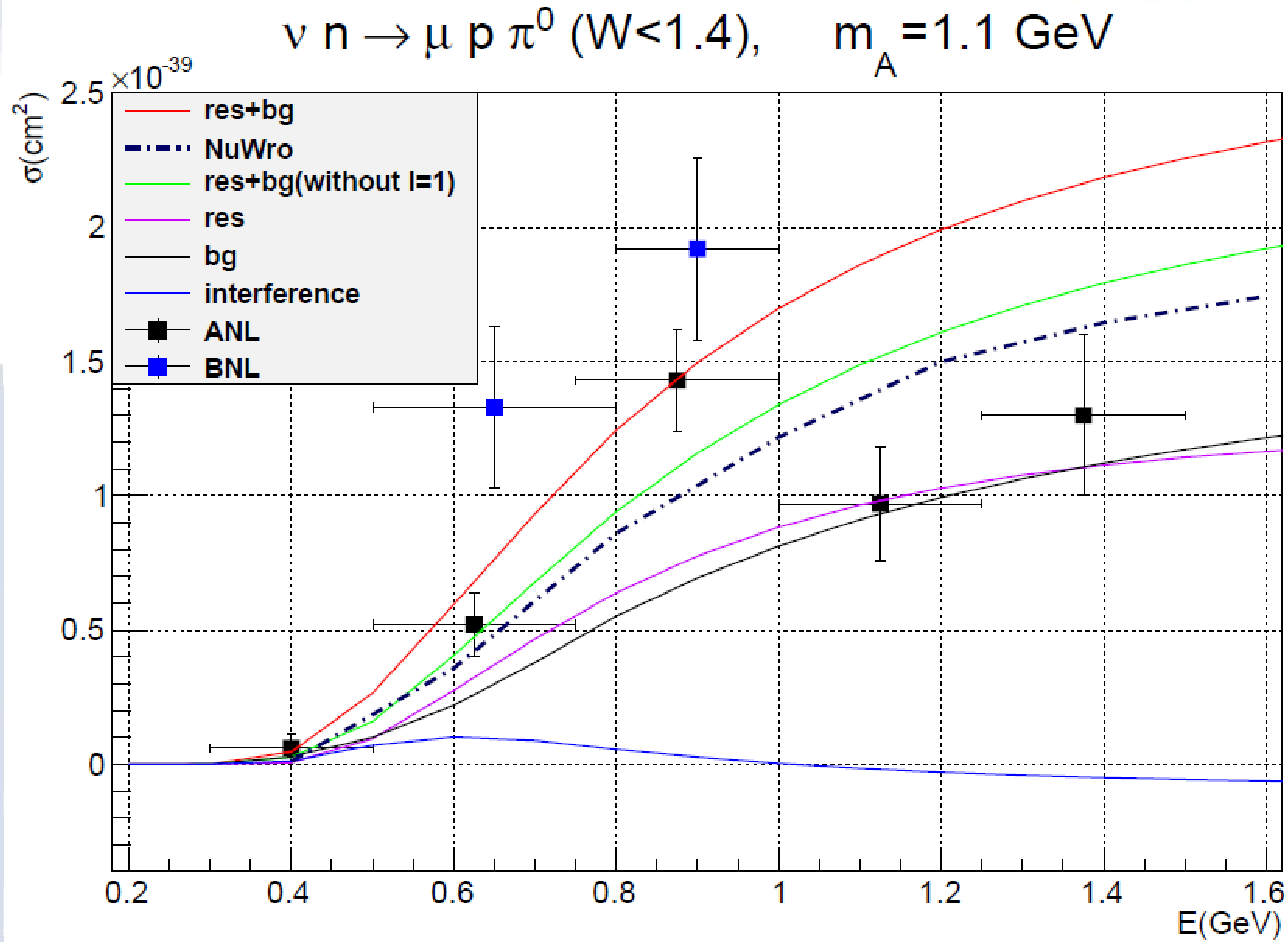
- Lepton mass has effect on phase space.
  This effect appear after integration.
- Lepton mass has effect on kinematics
- Lepton mass has effect on dynamics



ν p → μ p π⁺ (W<1.4), m_A =1.1 GeV
σ(cm²)
×10⁻³⁹
E(GeV)
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL
BNL



ν n → μ n π+ (W<1.4),   m_A =1.1 GeV
×10^-39
σ(cm^2)
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL
BNL
0
0.5
1
1.5
2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
E(GeV)



ν n → μ p π⁰ (W<1.4), m_A =1.1 GeV
×10^-39
σ(cm²)
2.5
2
1.5
1
0.5
0
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL
BNL
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
E(GeV)

Kinematics

$$\nu_l(k_1) + N(p_1) \to l(k_2) + N(p_2) + \pi(q)$$

$$\begin{aligned} k^2 &= (k_1 - k_2)^2 \\ W^2 &= (k + p_1)^2 = (q + p_2)^2 \end{aligned}$$

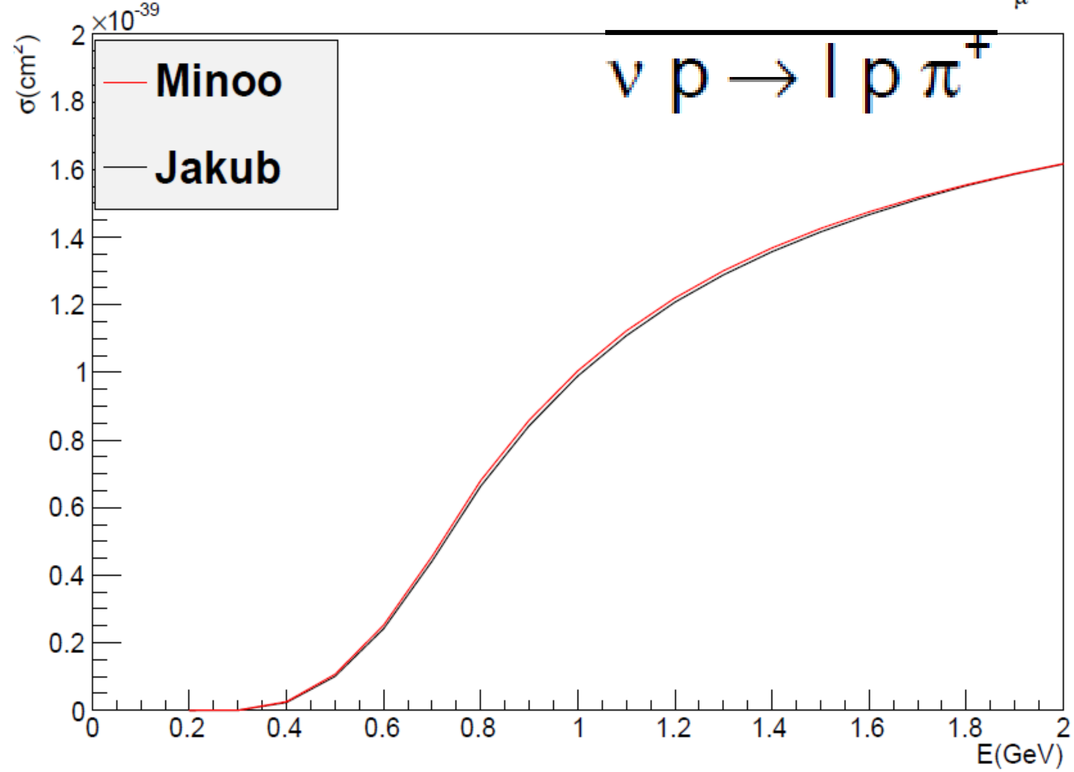
$$\nu_l(k_1) + N(p_1) \to l(k_2) + H(W, \mathbf{0})$$

$$\mathbf{q} + \mathbf{p_2} = \mathbf{k} + \mathbf{p_1} = \mathbf{0}$$

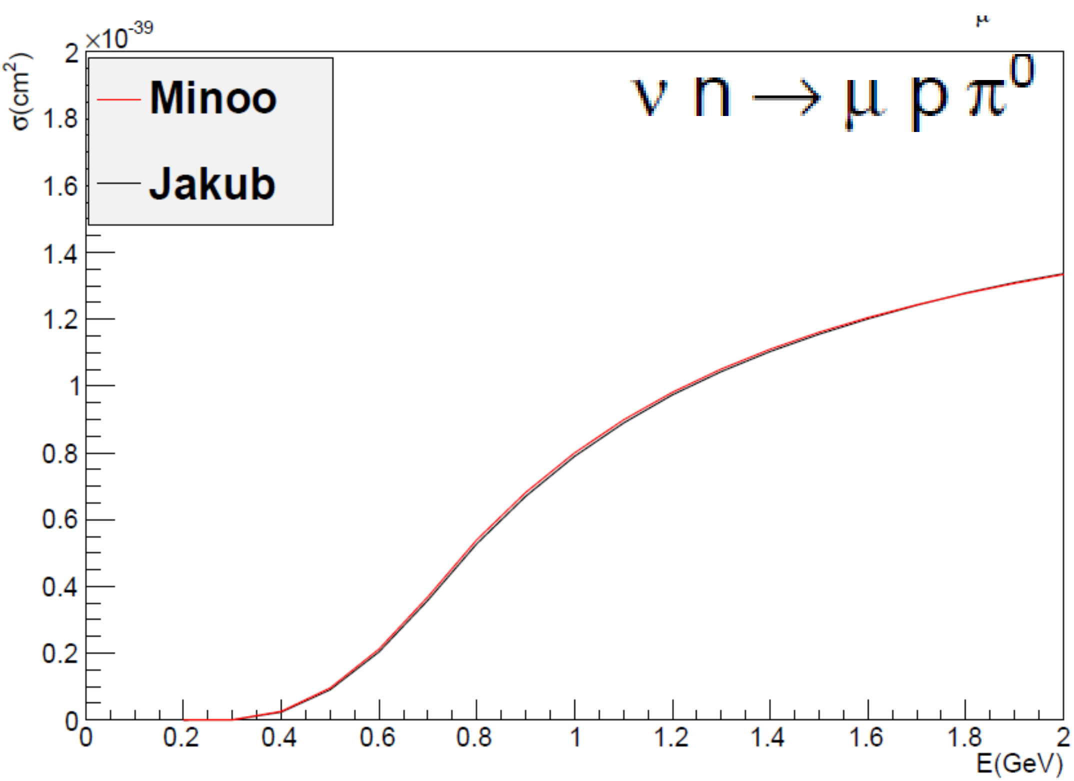
$$\begin{aligned} k_1 + p_1 &= k_2 + H \\ (H - k)^2 &= p_1^2 \\ W^2 + k^2 - 2Wk_0 &= M_N^2 \end{aligned}$$

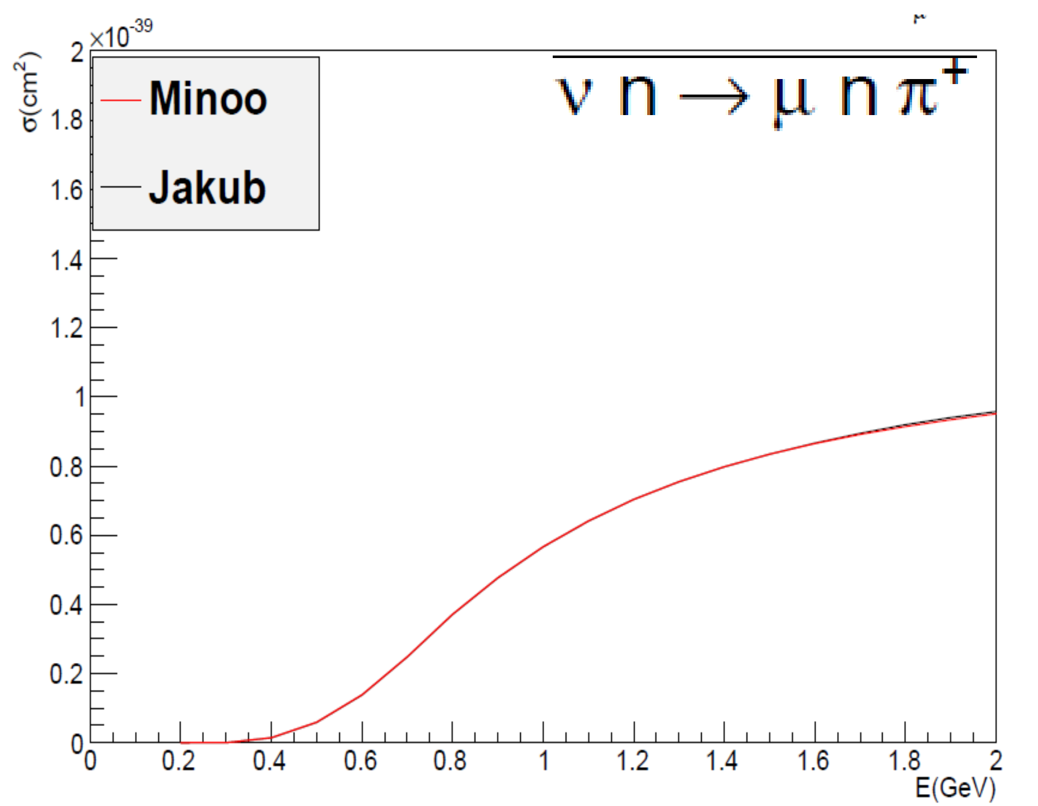
$$k_0 = \frac{W^2 + k^2 - M_N^2}{2W}$$

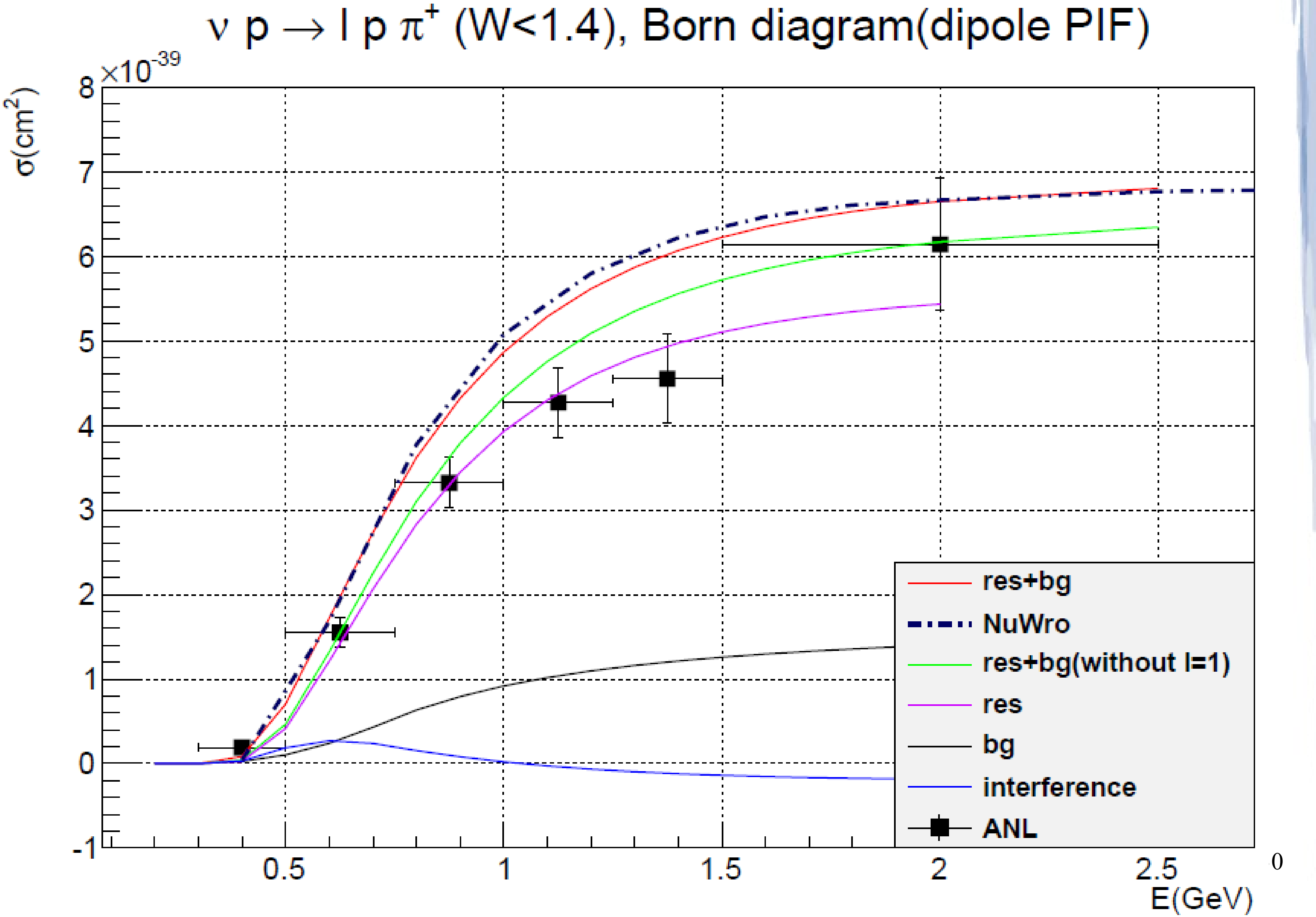
$$
\begin{aligned}
(k_1 + p_1)_L^2 &= (k_2 + H)^2 \\
(k_1^2 + p_1^2 + 2k_1 p_1)_L &= k_2^2 + H^2 + 2k_2 H \\
M_N^2 + 2k_{10} M_N &= m_l^2 + W^2 + 2W k_{20} \\
k_{20} &= \frac{M_N^2 + 2E_\nu M_N - W^2 - m_l^2}{2W} \\
k_{10} - k_{20} &= \frac{W^2 + k^2 - M_N^2}{2W} \\
k_{10} &= \frac{W^2 + k^2 - M_N^2 + 2W k_{20}}{2W} \\
u &= \frac{k_{10}^L + k_{20}^L + |\mathbf{k}^L|}{2k_{10}^L} = \frac{k_{10} + k_{20} + |\mathbf{k}|}{2k_{10}^L} . \frac{|\mathbf{k}^L|}{|\mathbf{k}|} \\
v &= \frac{k_{10}^L + k_{20}^L - |\mathbf{k}^L|}{2k_{10}^L} = \frac{k_{10} + k_{20} - |\mathbf{k}|}{2k_{10}^L} . \frac{|\mathbf{k}^L|}{|\mathbf{k}|}
\end{aligned}
$$



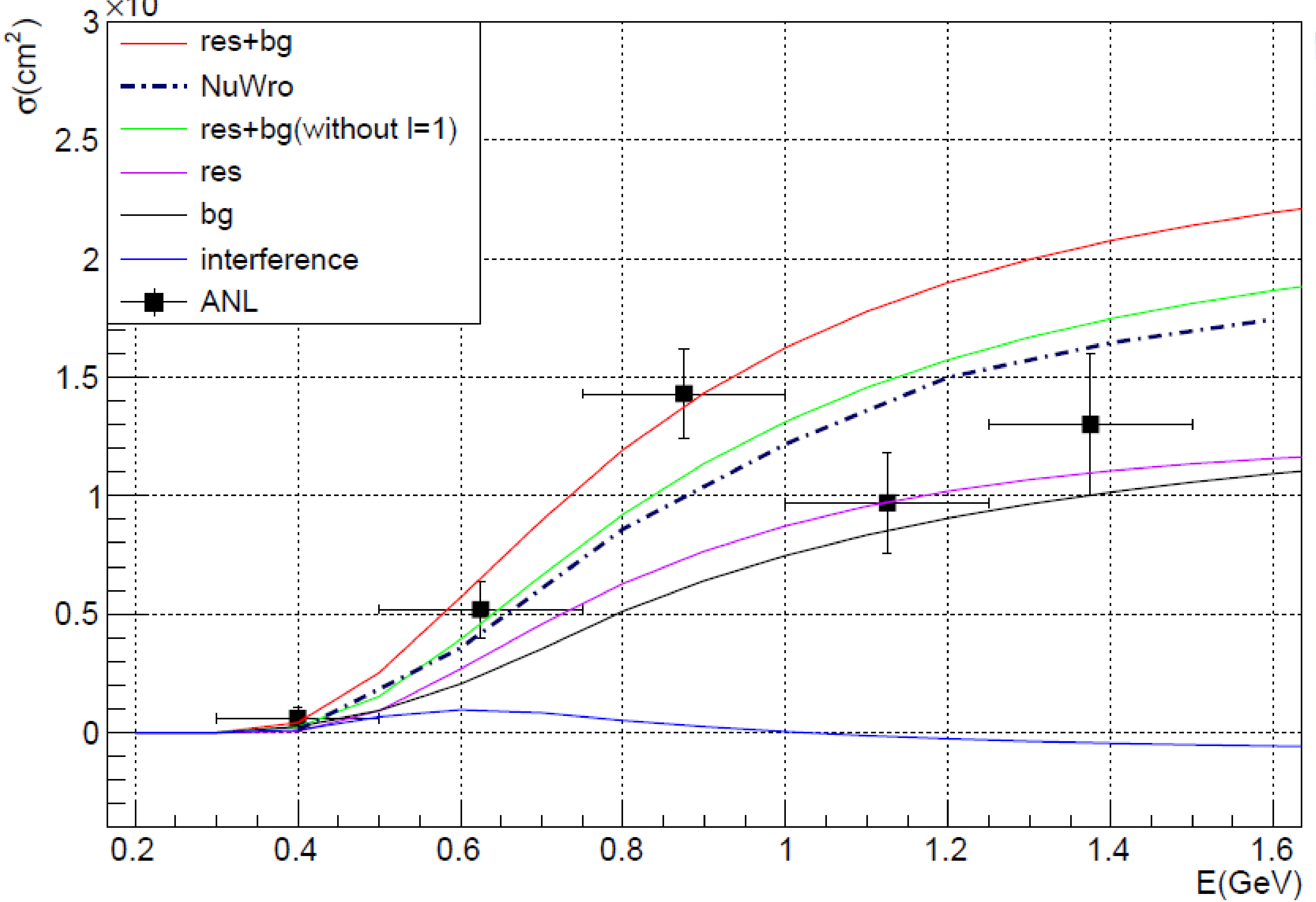
Comparison between
Jakub's result and mine
For **Born** diagrams



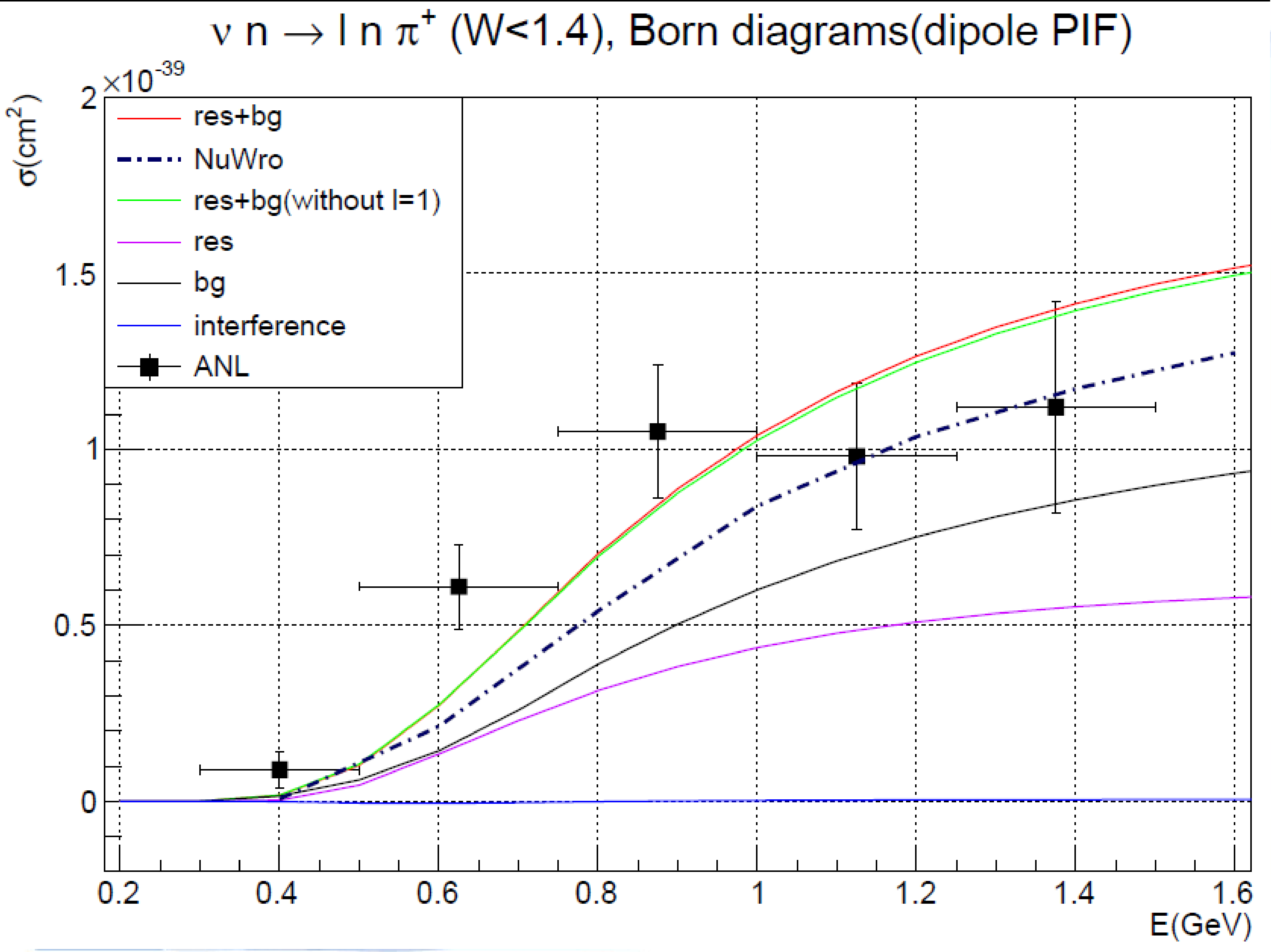




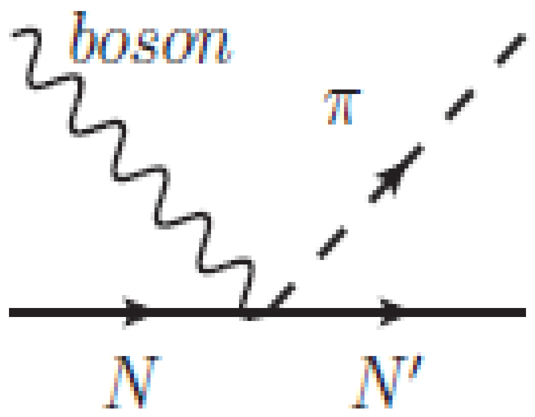
ν p → l p π+ (W<1.4), Born diagram(dipole PIF)
×10^-39
σ(cm²)
8
7
6
5
4
3
2
1
0
-1
0.5
1
1.5
2
2.5
E(GeV)
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL
0



×10
σ(cm²)
3
2.5
2
1.5
1
0.5
0
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
E(GeV)


ν n → l n π⁺ (W<1.4), Born diagrams(dipole PIF)
σ(cm²)
×10⁻³⁹
2
1.5
1
0.5
0
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
E(GeV)

Contact-Term ... the 4th diagram



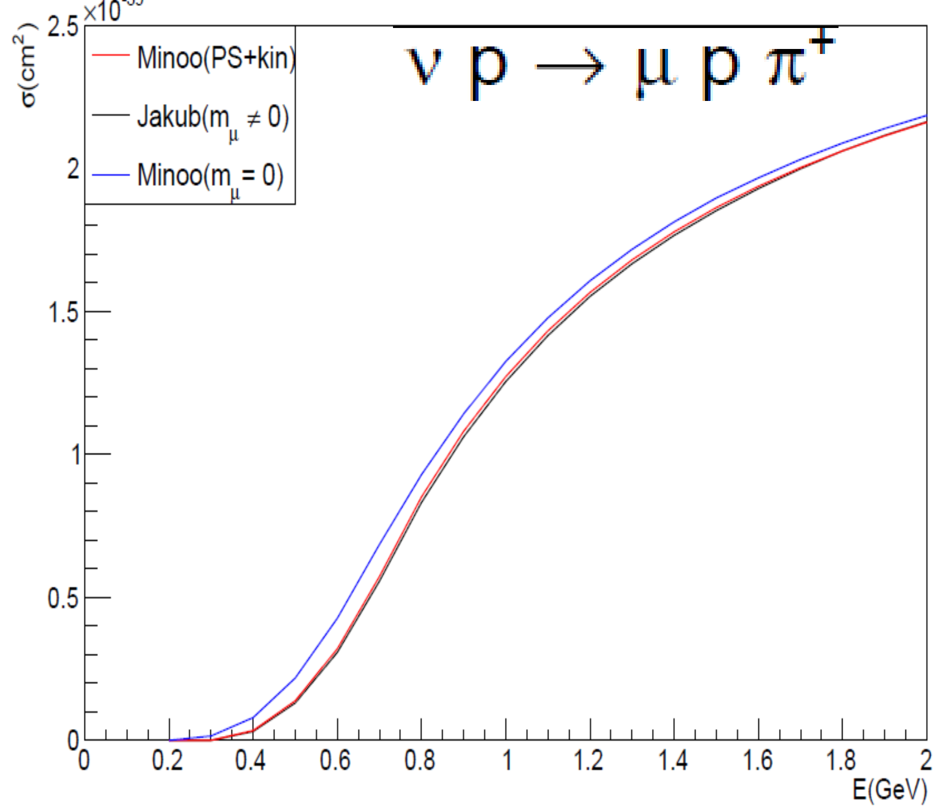
$$\mathcal{M} = \frac{G}{\sqrt{2}} cos\theta_C \ \bar{u}(p_2) \ [\sum_{k=1}^{6} V_k O(V_k) - \sum_{k=1}^{8} A_k O(A_K)] \ u(P_1)$$

$$\mathcal{M}_{CT} = -\frac{G}{\sqrt{2}} \frac{1}{f_\pi} \ cos\theta_C F_\rho((k-q)^2) \ \bar{u}(p_2) \epsilon^\mu \gamma_\mu u(P_1)$$

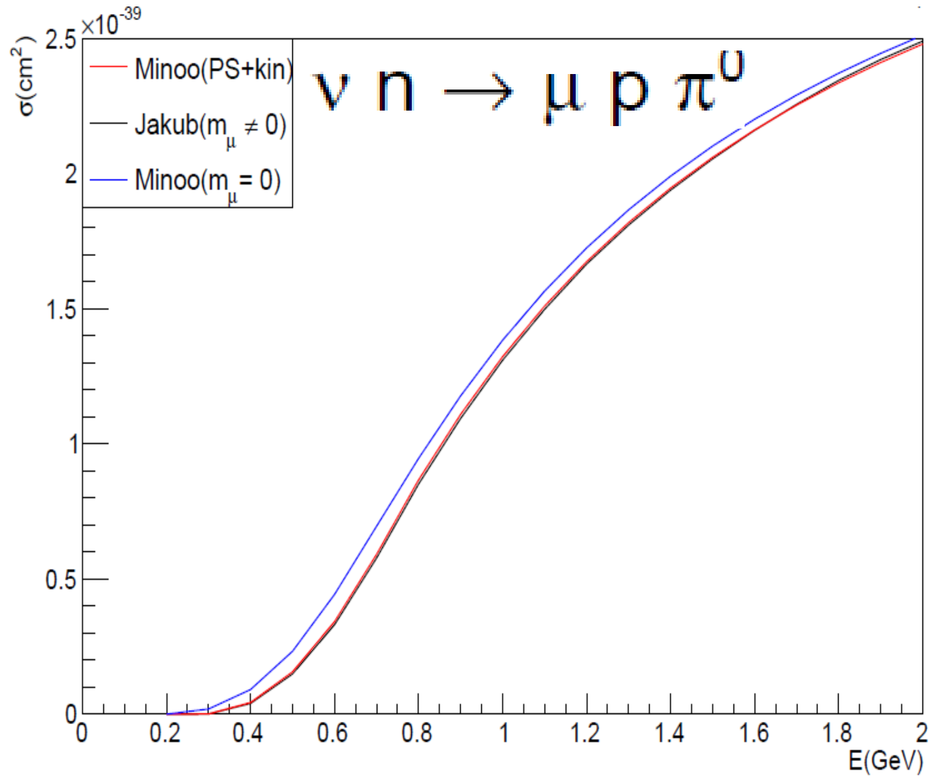
$$= -\frac{G}{\sqrt{2}} cos\theta_C \ \bar{u}(p_2) \ \sum_{k=1}^{8} A_k(s,t,u) O(A_K) \ u(P_1)$$

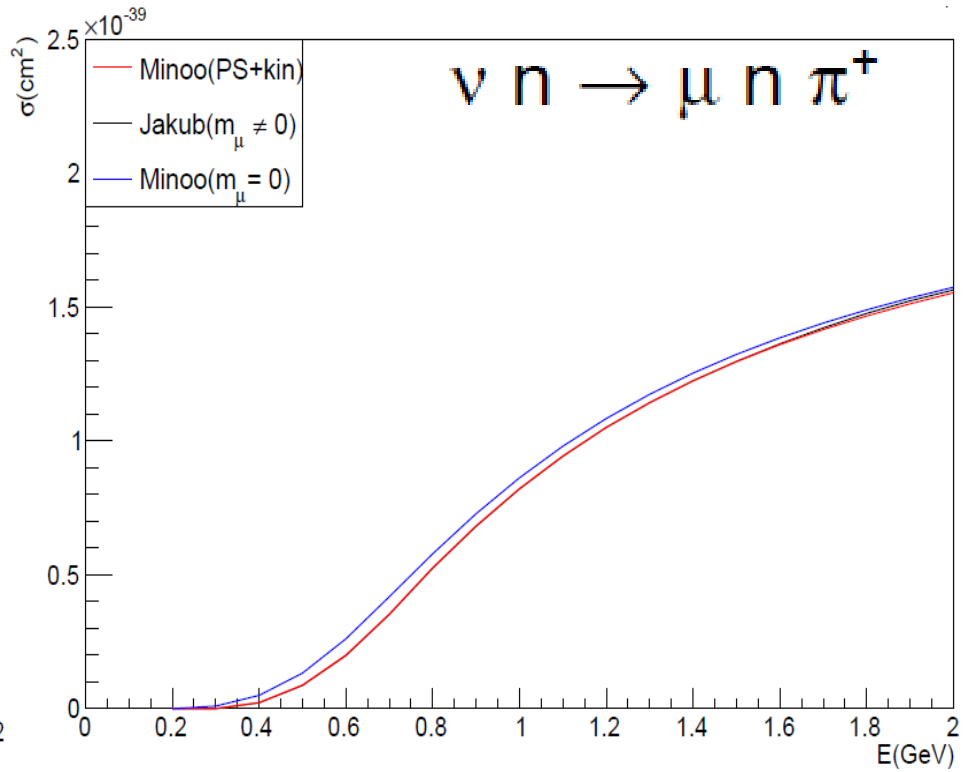
$$O(A_4) = M(\epsilon^\mu \gamma_\mu)$$

$$A_4 = \frac{1}{f_\pi} \frac{1}{M} F_\rho((k-q)^2)$$

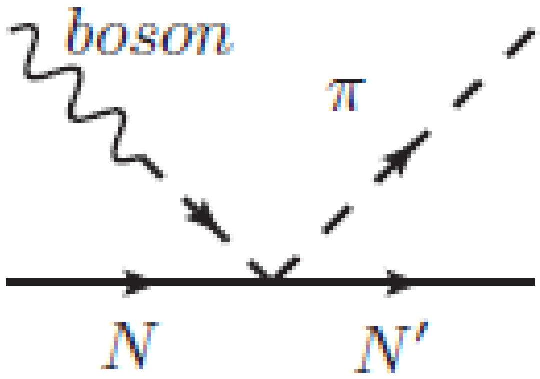


Comparison between **massive** and **massless** lepton for **4_diagrams**





Pion_Pole diagram



$$\mathcal{M} = \frac{G}{\sqrt{2}} cos\theta_C \; \bar{u}(p_2) \; [\sum_{k=1}^{6} V_k O(V_k) - \sum_{k=1}^{8} A_k O(A_K)] \; u(P_1)$$

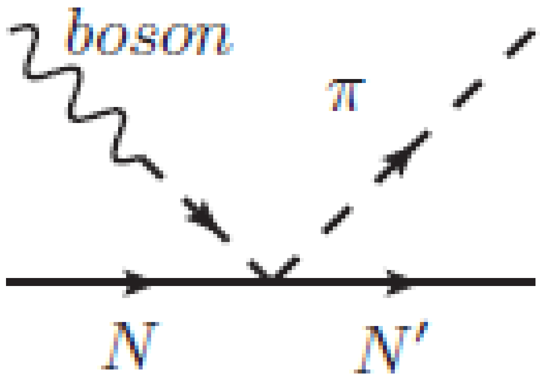
$$\mathcal{M}_{PP} = -\frac{G}{\sqrt{2}} \frac{1}{f_\pi} cos\theta_C F_\rho((k-q)^2) \; \frac{1}{k^2 + m_\pi^2} \; \bar{u}(p_2) \epsilon^\mu k_\mu \; \not{k} u(P_1)$$

$$= -\frac{G}{\sqrt{2}} cos\theta_C \; \bar{u}(p_2) \; \sum_{k=1}^{8} A_k(s,t,u) O(A_K) \; u(P_1)$$

$$O(A_8) = - \not{k}(\epsilon^\mu k_\mu)$$

$$A_8 = -\frac{1}{f_\pi} \; F_\rho((k-q)^2) \; \frac{1}{k^2 + m_\pi^2} \bar{u}(p_2)$$

Pion_Pole diagram



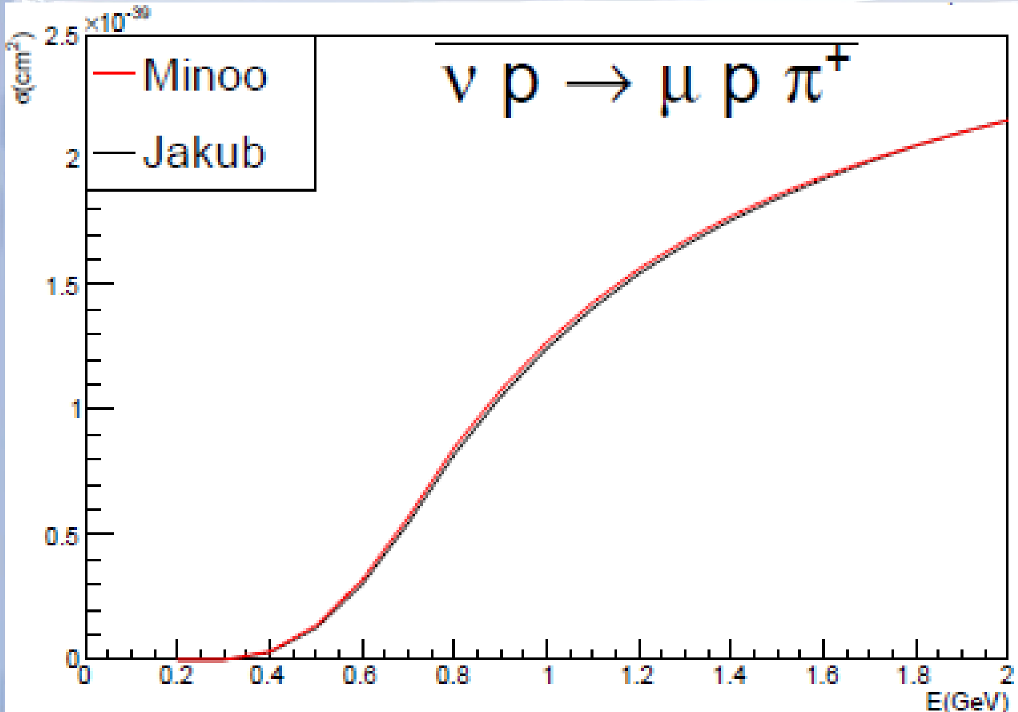
$$\mathcal{M} = \frac{G}{\sqrt{2}} cos\theta_C \; \bar{u}(p_2) \; [\sum_{k=1}^{6} V_k O(V_k) - \sum_{k=1}^{8} A_k O(A_K)] \; u(P_1)$$

$$\mathcal{M}_{PP} = -\frac{G}{\sqrt{2}} \frac{1}{f_\pi} cos\theta_C F_\rho((k-q)^2) \; \frac{1}{k^2 + m_\pi^2} \; \bar{u}(p_2) \epsilon^\mu k_\mu \; \not{k} u(P_1)$$

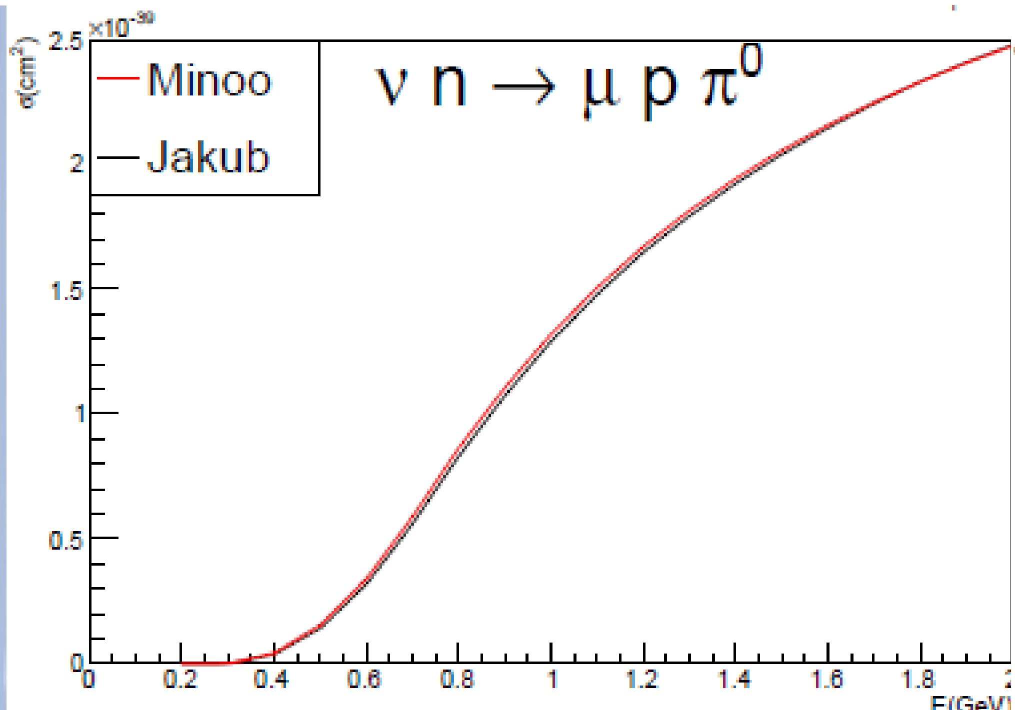
$$= -\frac{G}{\sqrt{2}} cos\theta_C \; \bar{u}(p_2) \; \sum_{k=1}^{8} A_k(s,t,u) O(A_K) \; u(P_1)$$

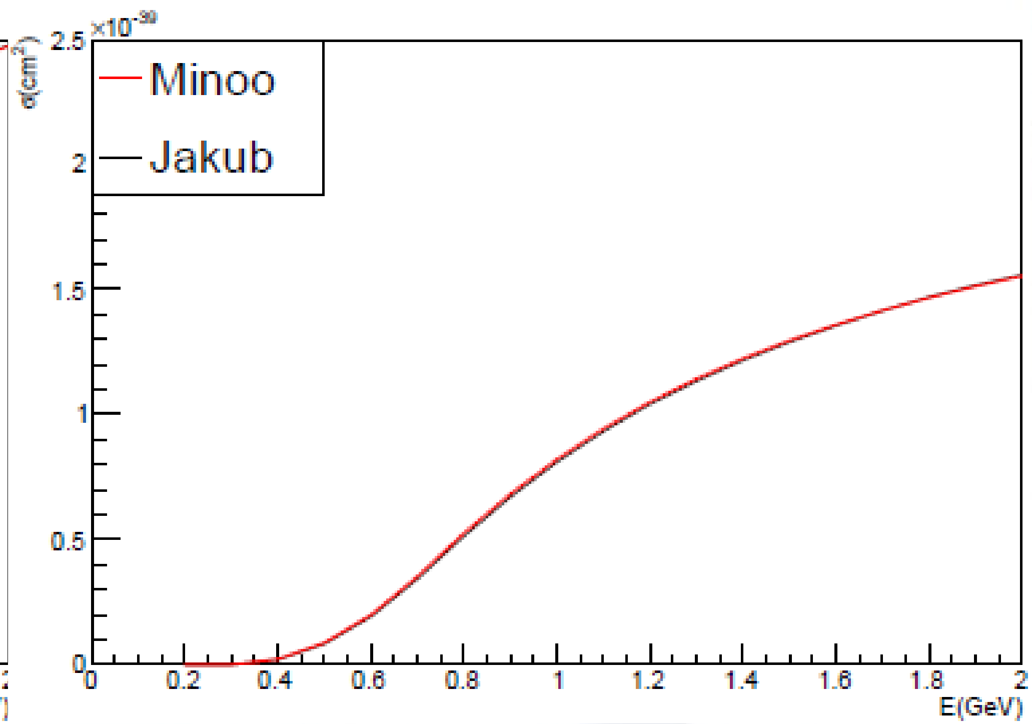
$$O(A_8) = - \; \not{k}(\epsilon^\mu k_\mu)$$

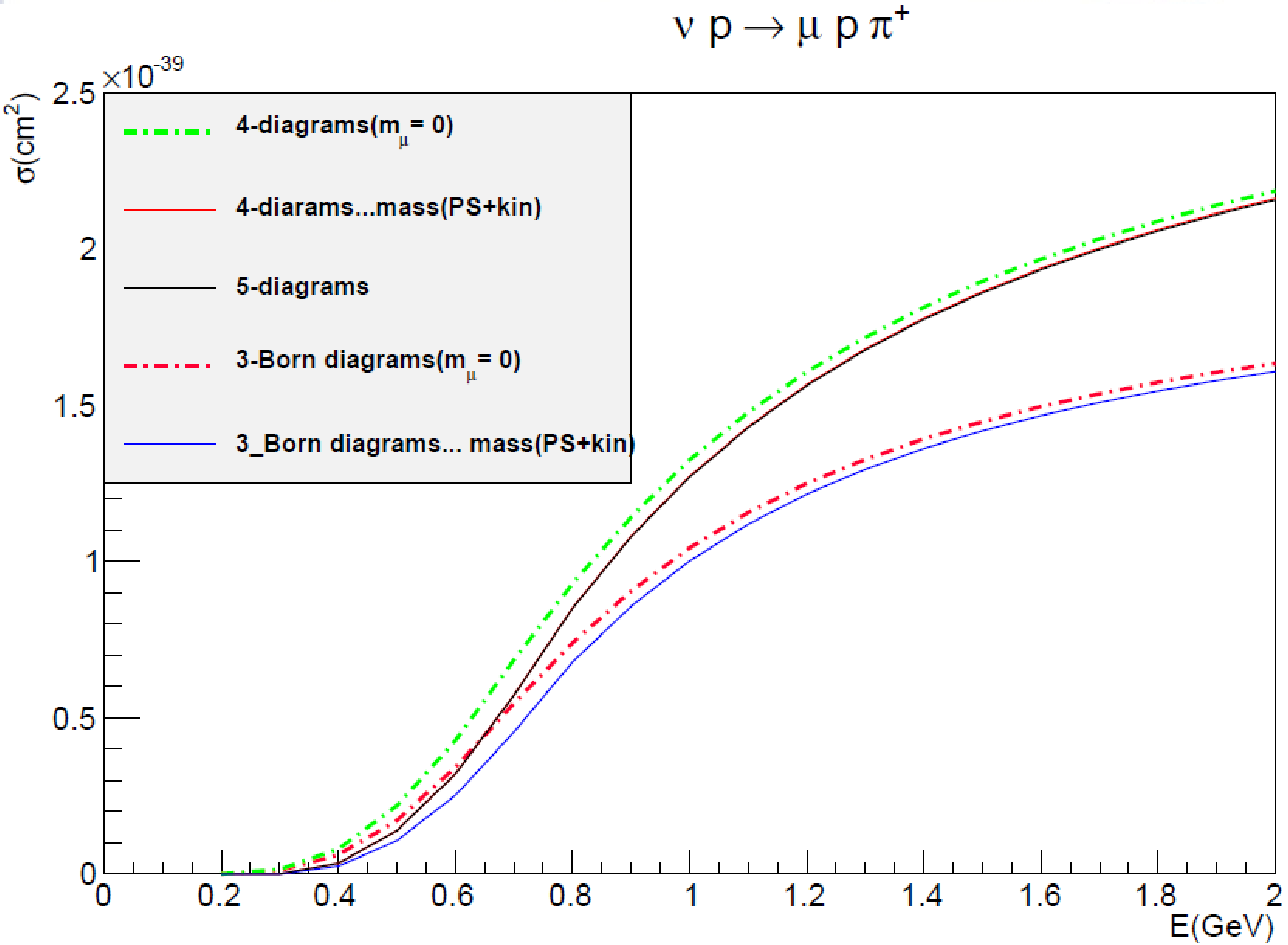
$$A_8 = -\frac{1}{f_\pi} \; F_\rho((k-q)^2) \; \frac{1}{k^2 + m_\pi^2} \bar{u}(p_2)$$



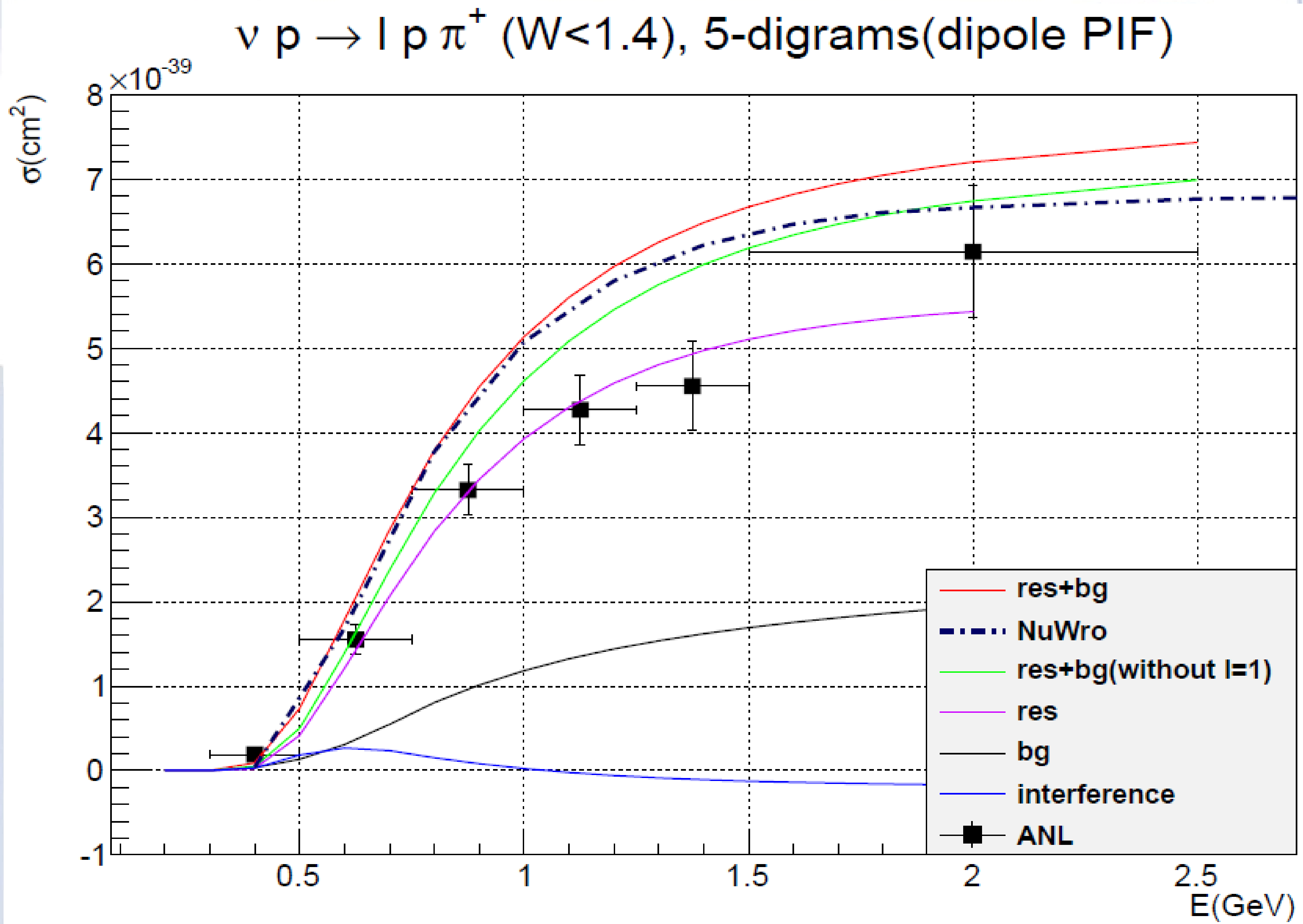
Comparison between Jakub's result and mine for **5-diagrams** after mass correction



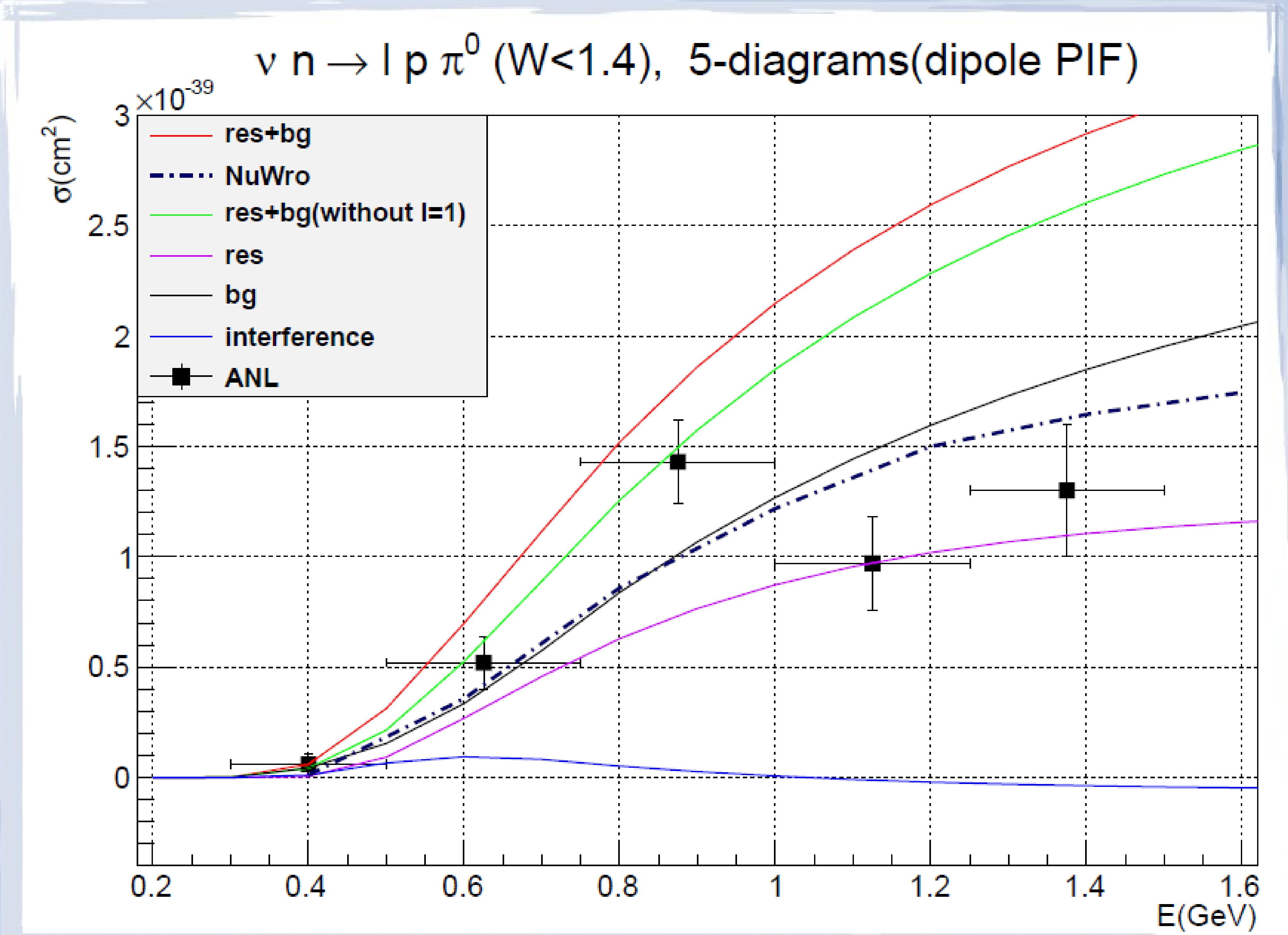




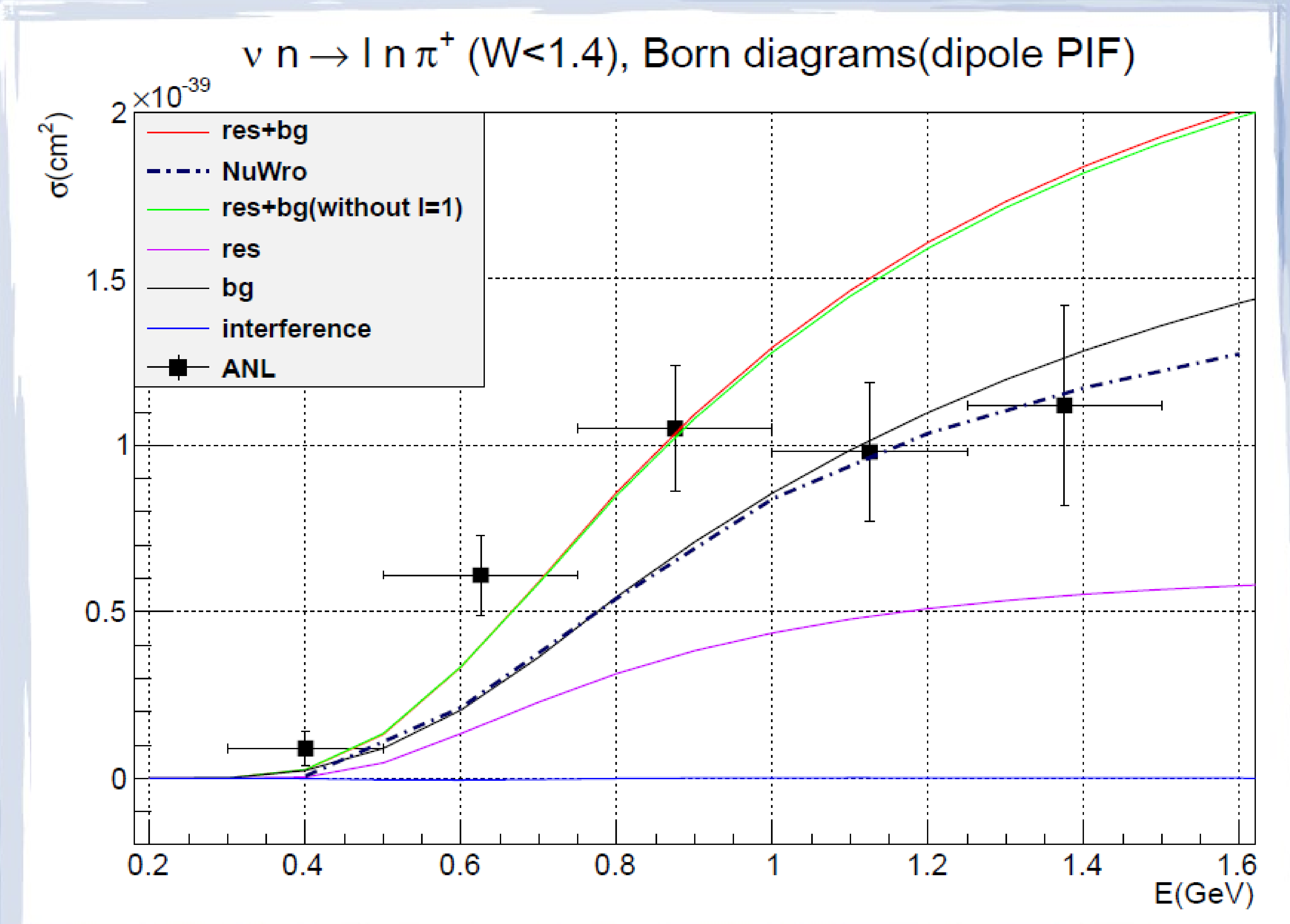
ν p → μ p π+
σ(cm2)
×10-39
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
E(GeV)
4-diagrams(mμ = 0)
4-diarams...mass(PS+kin)
5-diagrams
3-Born diagrams(mμ = 0)
3_Born diagrams... mass(PS+kin)



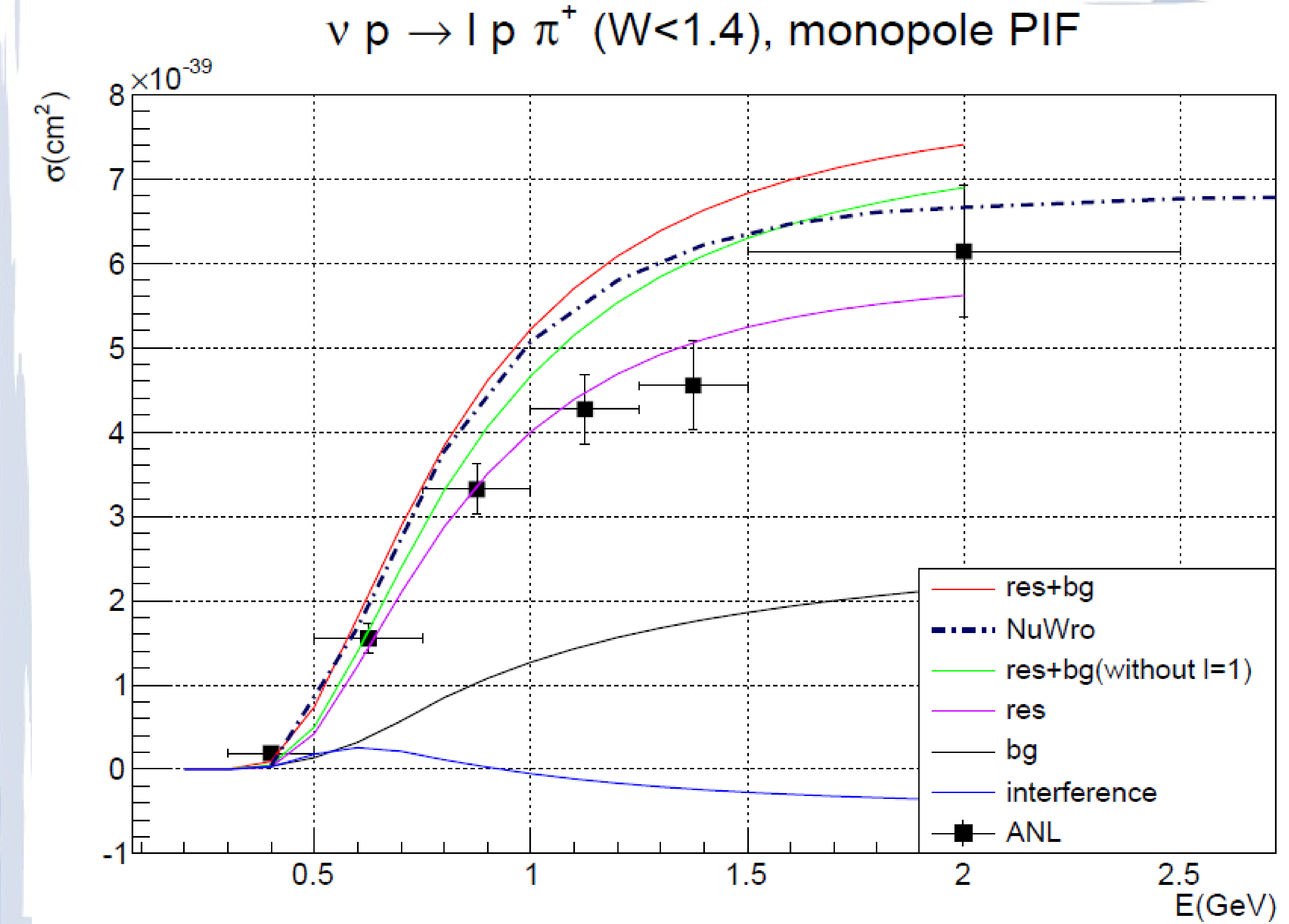
ν p → l p π+ (W<1.4), 5-digrams(dipole PIF)
×10-39
σ(cm2)
8
7
6
5
4
3
2
1
0
-1
0.5
1
1.5
2
2.5
E(GeV)
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL



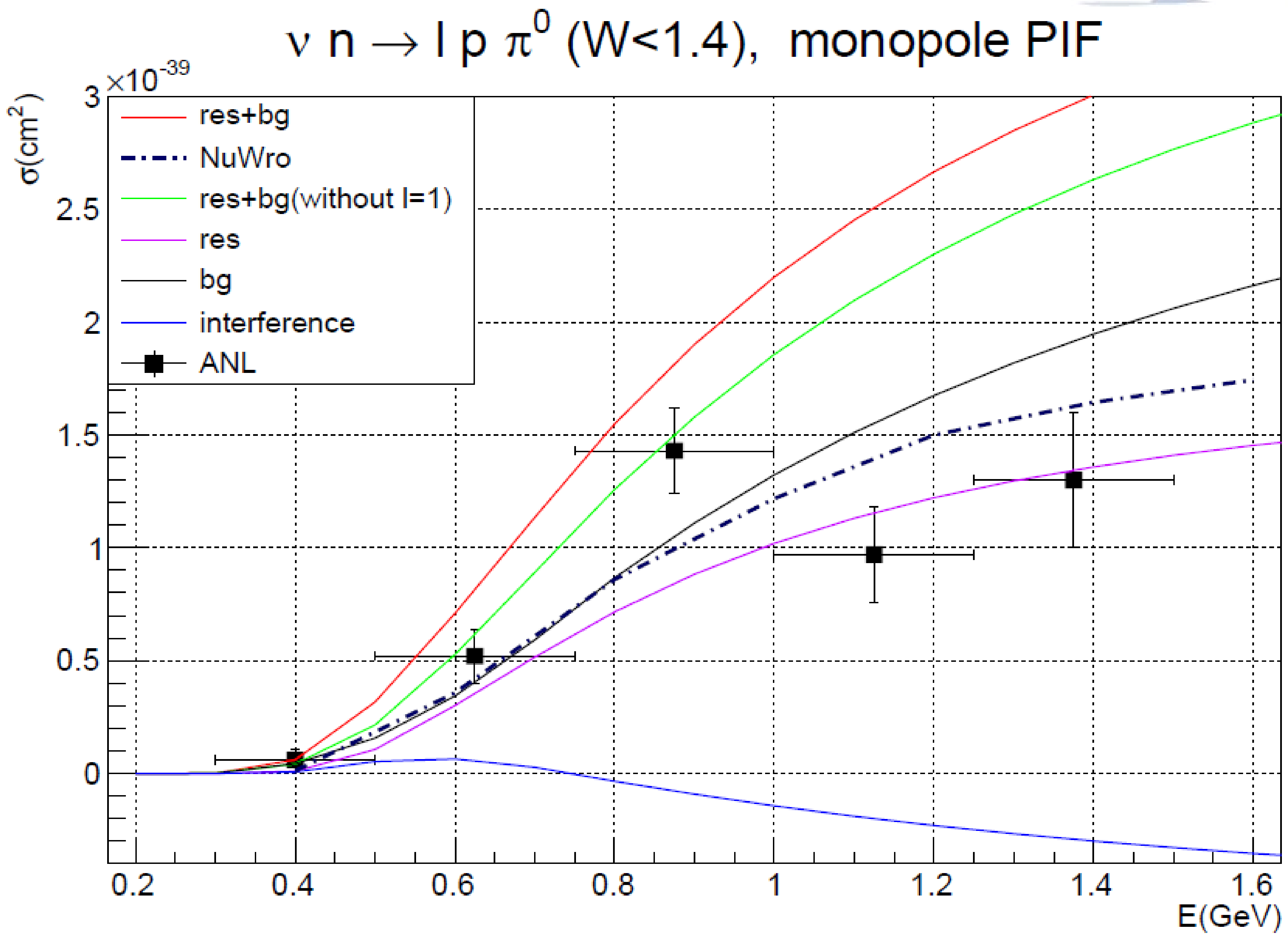
$\nu\ n \to l\ p\ \pi^0$ (W<1.4), 5-diagrams(dipole PIF)
σ(cm²)
×10⁻³⁹
3
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
E(GeV)
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL



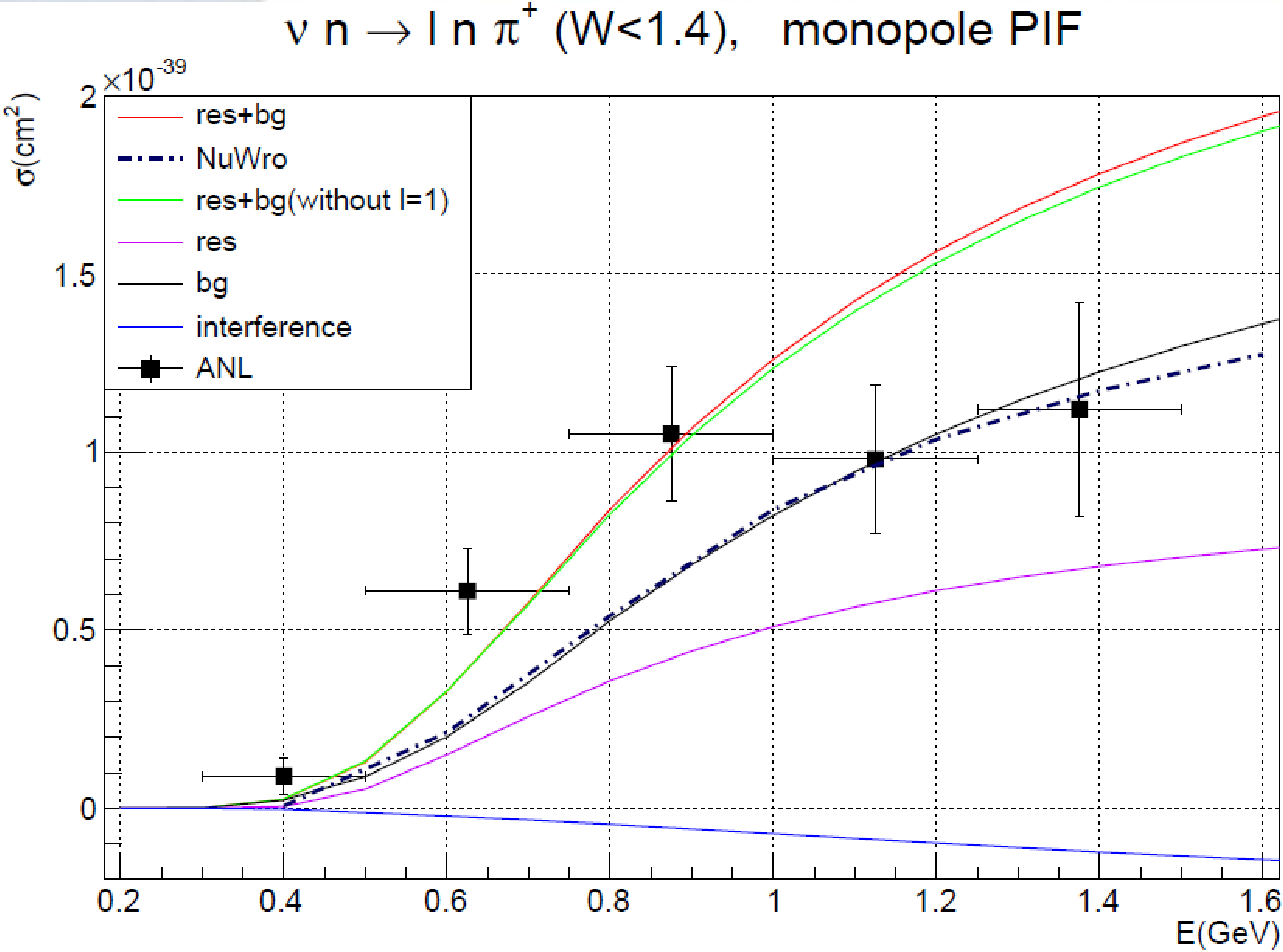
ν n → l n π+ (W<1.4), Born diagrams(dipole PIF)
×10-39
σ(cm2)
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
E(GeV)



ν p → l p π+ (W<1.4), monopole PIF
σ(cm2)
×10-39
8
7
6
5
4
3
2
1
0
-1
0.5
1
1.5
2
2.5
E(GeV)
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL



ν n → l p π⁰ (W<1.4), monopole PIF
σ(cm²)
×10⁻³⁹
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL
E(GeV)
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
0.5
1
1.5
2
2.5
3



ν n → l n π+ (W<1.4), monopole PIF
×10^-39
σ(cm^2)
res+bg
NuWro
res+bg(without I=1)
res
bg
interference
ANL
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
E(GeV)