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Statistical model in the description of particle production in heavy-ion collisions

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Statistical Hadronization Model (SHM)

Particles are produced in accordance with their (statistical) phase space densities.

E.Fermi, Prog. Theor. Phys. **5**, 570 (1950)

When decays of resonances are included, the SHM describes quantitatively the (relative) yields of measured particles in heavy-ion collisions.

Hydrodynamical Evolution

"At the moment of a collision the large amount of particles is created, concentrated in the volume of the size determined by the range of nuclear forces and collision energy. With time passing, this system expands,... a part of the expansion should be hydrodynamical."

L. D. Landau, Izv. Akad. Nauk Ser. Fiz. **17**, 51 (1953)

$z = 0$ plane

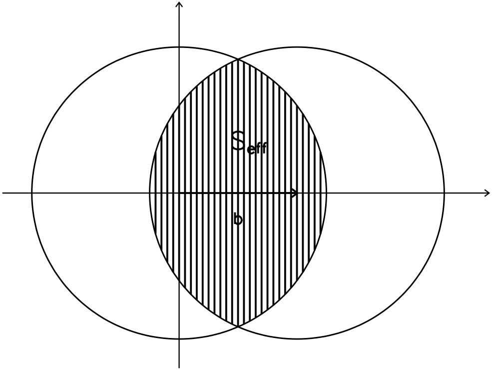
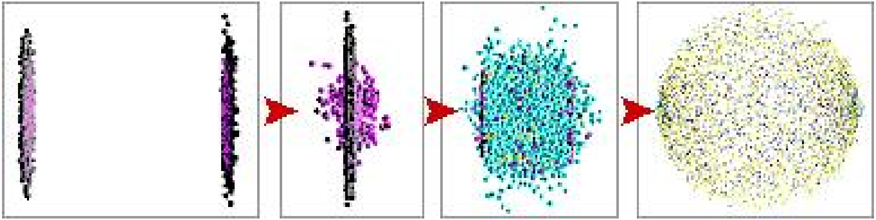
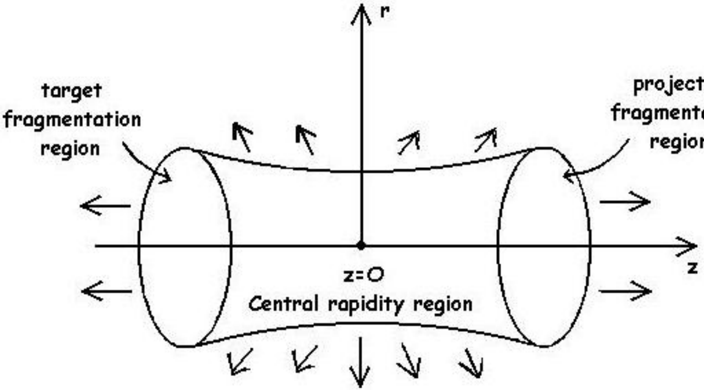


Figure: View of an AA collision at impact parameter b. The region where the nuclei overlap has been hatched and its area equals S_{eff}.

Steps of a central collision



A central collision



Hydrodynamic equations of the perfect fluid

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

$$u^\mu = \gamma(1, \vec{v}) \;, \qquad \gamma = \frac{1}{\sqrt{1 - v^2}}$$

How (when) the statistical system ends?

Generally, the freeze-out moment (the end of a statistical system) is defined as the moment when hadrons cease to interact and start to stream freely to detectors.

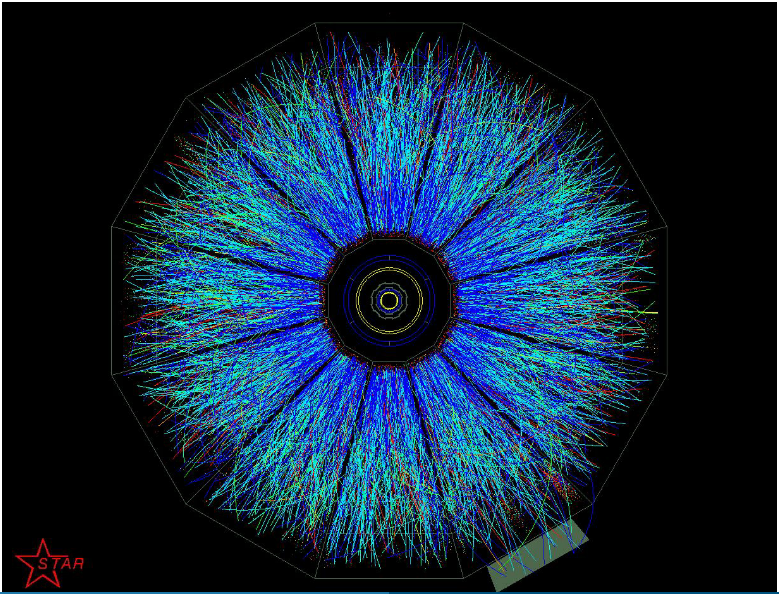
The whole experimental information we get (the data) is from this particular moment, this is like the photo taken at this (and only this) moment.

We model the freeze-out by imposing the condition:

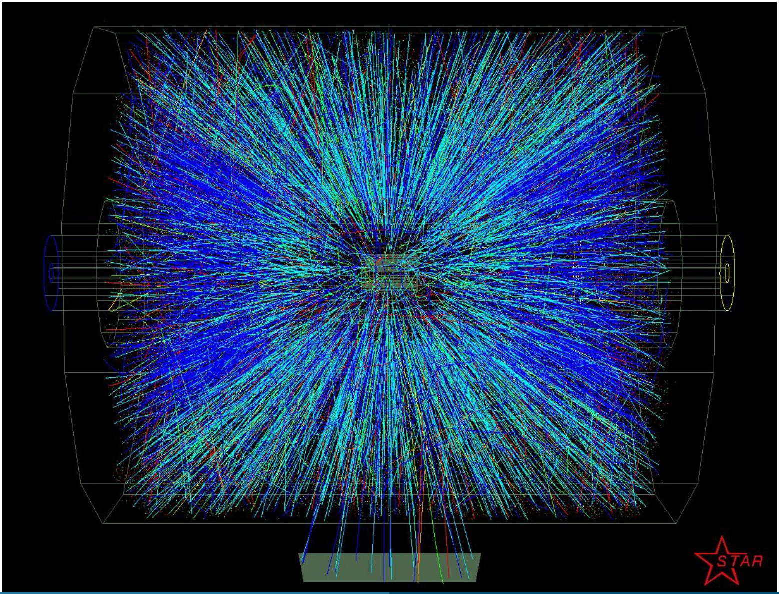
$$T(\vec{r},t) = T_{f.o.} = constant \ .$$

This defines the 3dim freeze-out hypersurface.

Front view of an Au-Au collision



Side view of an Au-Au collision



What does SHM mean?

For the boost invariant system:

$$\boxed{\frac{(dN_i/dy)_{y=0}}{(dN_j/dy)_{y=0}} = \frac{N_i}{N_j} = \frac{n_i}{n_j}}, \qquad y = \frac{1}{2}\ln\frac{E+p_L}{E-p_L}$$

$$n_i(T,\mu_B) = n_i^{prim}(T,\mu_B) + \sum_a \varrho(i,a)\, n_a^{prim}(T,\mu_B)\ ,$$

$n_i^{prim}(T,\mu_B)$ - the thermal density of particle species i at the freeze-out

$\varrho(i,a)$ - the final number of particle species i which can be received from all possible decays (cascades) of particle a, the sum is over all kinds of resonances in the hadron gas

Primordial distributions

At the freeze-out the momentum distributions are frozen and these are primordial distributions:

$$f_i^{prim} = \frac{(2s_i + 1)}{(2\pi\hbar c)^3} \frac{1}{\exp\left\{\frac{E_i - \mu_i}{T}\right\} \pm 1}$$

$$\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$$

$$n_i^{prim} = \int d\vec{p}\, f_i^{prim}(\vec{p})$$

$$\sum S_i n_i = 0\ , \qquad \frac{\sum Q_i n_i}{\sum B_i n_i} = \frac{Z}{A}$$

The minimization of χ^2 function

$$\chi^2(\alpha_1, ..., \alpha_l) = \sum_{k=1}^{n} \frac{(R_k^{exp} - R_k^{th}(\alpha_1, ..., \alpha_l))^2}{\sigma_k^2} \ ,$$

R_k^{exp} - the kth measured quantity,

$R_k^{th}(\alpha_1, ..., \alpha_l)$ - its theoretical prediction,

σ_k - the error of the kth measurement,

$n_{dof} = n - l$ - the number of degrees of freedom.

$$\frac{\chi^2_{min}}{n_{dof}} \sim 1$$

Fit to the ratios: Au-Au at $\sqrt{s_{NN}} = 130$ GeV

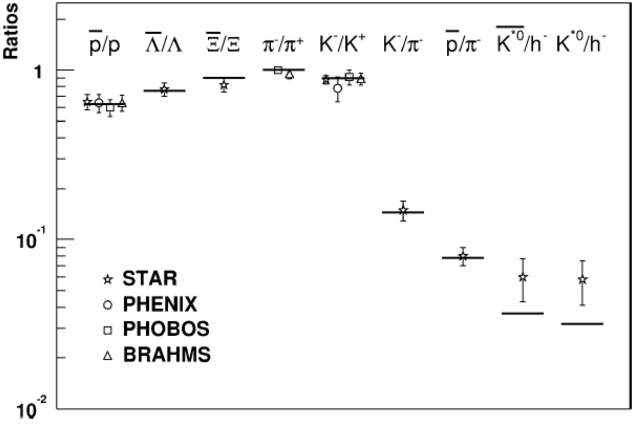


Fig. 2. Comparison between RHIC experimental particle ratios and statistical model calculations with $T = 174$ MeV and $\mu_B = 46$

P. Braun-Munzinger *et al.*, Phys.Lett.**B518** (2001) 41

Thermal distributions without the flow

The total multiplicity of particle species *i*:

$$N_i = V \int d\vec{p}\, f_i(\vec{p}) = V \int dy \int dp_T (2\pi p_T) E_i f_i = \int dy \int dp_T \; \frac{d^2 N_i}{dp_T dy}$$

The transverse momentum distribution at a given rapidity:

$$\frac{d^2 N_i}{2\pi p_T dp_T dy} = V E_i \cdot f_i = A\, m_{T,i} \cosh(y) \exp\left\{ - \frac{m_{T,i}\cosh(y) - \mu_i}{T} \right\}$$

$$m_{T,i} = \sqrt{m_i^2 + p_T^2}, \quad E_i = m_{T,i}\cosh(y), \qquad y = \frac{1}{2}\ln\frac{E + p_L}{E - p_L}$$

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The Cooper-Frye formula

$$dN = \int_\sigma f(x,p)\, d\vec{p}\, \frac{p^\mu}{E}\, d\sigma_\mu$$

the total number of particles with momenta in $[\vec{p}, \vec{p} + d\vec{p}]$ emitted (decoupled) from the hypersurface σ^μ,
$d\sigma_\mu$ is the normal vector to the hypersurface.

$$\boxed{E\,\frac{dN}{d^3p} = \frac{dN}{d^2p_T\, dy} = \int_\sigma f(x,p)\, p^\mu\, d\sigma_\mu}$$

F.Cooper, G.Frye and E.Schonberg, Phys.Rev.**D11**, 192 (1975)

The Cooper-Frye formula, *cont.*

$$\sigma^\mu = \sigma^\mu(\alpha, \eta, \phi) \quad - \quad \text{a freeze-out hypersurface}$$

$$j^\mu \quad - \quad \text{a particle density current} = \text{a fluid 4-flow}$$

$dQ = j^\mu \, d\sigma_\mu$ - the amount of the fluid (the number of particles) passing through the hypersurface element $d\sigma_\mu$

$$d\sigma_\mu = \epsilon_{\mu\nu\beta\gamma} \frac{\partial \sigma^\nu}{\partial \alpha} \frac{\partial \sigma^\beta}{\partial \eta} \frac{\partial \sigma^\gamma}{\partial \phi} \, d\alpha \, d\eta \, d\phi$$

$j^\mu = f(x,p) \, d\vec{p} \, \frac{p^\mu}{E}$ - the particle density current with momenta in $[\vec{p}, \vec{p} + d\vec{p}]$

The distribution in the presence of the flow

$$f_i(\vec{r}, \vec{q}, t) = \frac{(2s_i + 1)}{(2\pi\hbar c)^3} \frac{1}{\exp\left\{\frac{q_\nu u^\nu(\vec{r},t) - \mu_i(\vec{r},t)}{T(\vec{r},t)}\right\} \pm 1}$$

The Cracow Model

W. Broniowski and W. Florkowski, PRL **87** (2001) 272302; PRC **65** (2002) 064905; APPB **33** (2002) 1935

1. The freeze-out hypersurface and the Hubble-like expansion

$$\tau = \sqrt{t^2 - r_x^2 - r_y^2 - r_z^2} = const, \qquad u^\mu = \frac{x^\mu}{\tau}$$

   with condition $r = \sqrt{r_x^2 + r_y^2} < \rho_{max}$.

2. Contributions from resonance decays to the measured particle multiplicities and momentum distributions are taken into account completely.

Parameters of the Cracow Model

- Statistical parameters $T, \ \mu_B$
- Geometric parameters $\tau, \ \rho_{max}$
- All four parameters T, μ_B, ρ_{max} and τ are fitted to the spectra simultaneously in this version of the model [DP, APPB **40**, 2825 (2009)].

The invariant distribution of particle species i

$$\frac{dN_i}{d^2p_T\, dy} = \int p^\mu d\sigma_\mu\, f_i(p \cdot u)$$

f_i - final momentum distribution of the ith particle, *i.e.* with contributions from resonance decays:

$$f_i = f_i^{prim} + \sum_{decay} f_i^{decay}$$

RHIC - Relativistic Heavy Ion Collider

BNL - Brookhaven National Laboratory, Long Island, USA

RHIC accelerator complex

In the RHIC channel

Some technical details

circumference: 3.9 km, diameter: 1.2 km, 3.7 m underground

maximal beam velocity: 0.99995 c

beam: 57 bunches, billions of ions each

proton energy (or per nucleon): 100 GeV

energy of Au nucleus: 197×100 GeV ≈ 1 g dropped from h=0.3 mm but in the volume $\sim 10^{33}$ times smaller!

temperature of magnets = -268.7°C (+4.5°K)

Pion spectra at midrapidity

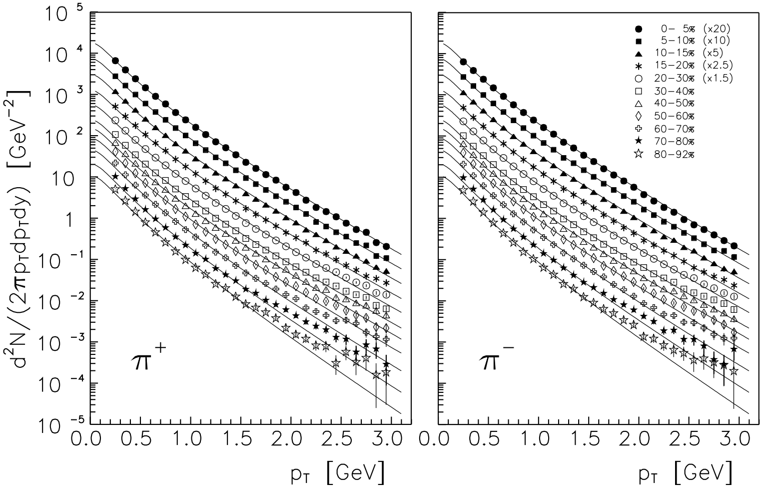

Figure: Invariant yields of π^+ (left) and π^- (right) as a function of p_T in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

Kaon spectra at midrapidity

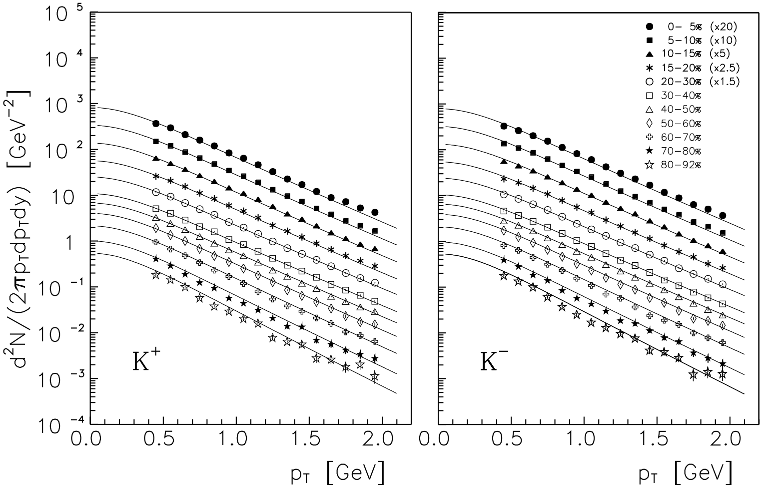

Figure: Invariant yields of K^+ (left) and K^- (right) as a function of p_T in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

Proton and antiproton spectra at midrapidity

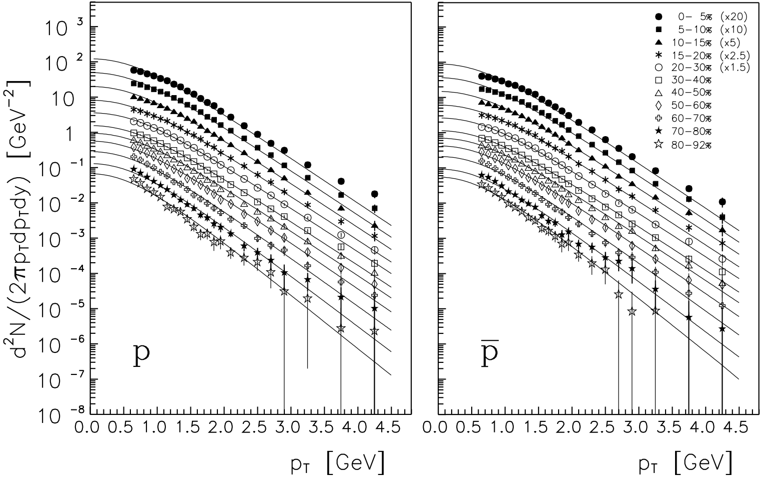

Figure: Invariant yields of protons (left) and antiprotons (right) as a function of p_T in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

PHOBOS spectra at very low p_T

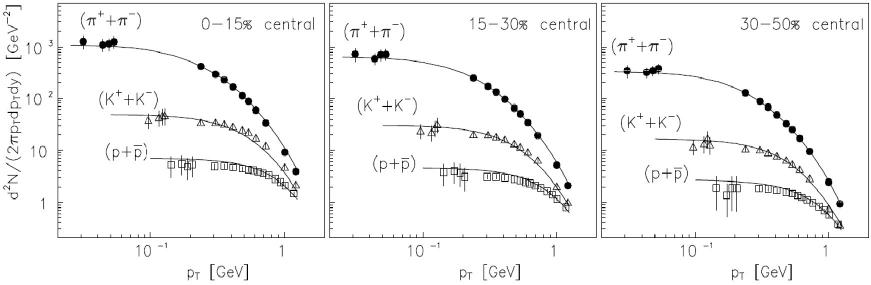

Figure: Invariant yields as a function of p_T in Au+Au collisions at $\sqrt{s_{NN}} = 62.4$ GeV.

PHENIX @ $\sqrt{s_{NN}} = 200$ GeV

PRC **69**, 034909 (2004)

Centr. [%]	T [MeV]	μ_B [MeV]	ρ_{max} [fm]	τ [fm]	$\beta_\perp^{max}$	$\chi^2/$ NDF
0-5	150.1±1.3	24.1±3.7	9.28±0.21	9.48±0.19	0.70	0.69
5-10	150.2±1.4	23.5±3.7	8.75±0.20	8.80±0.18	0.70	0.50
10-15	150.2±1.4	22.8±3.7	8.25±0.19	8.20±0.17	0.71	0.37
15-20	150.0±1.4	22.4±3.7	7.80±0.18	7.69±0.16	0.71	0.37
20-30	149.6±1.3	24.0±3.5	7.13±0.16	6.96±0.14	0.72	0.45
30-40	149.8±1.4	23.8±3.6	6.14±0.14	6.03±0.12	0.71	0.66
40-50	148.5±1.4	22.5±3.7	5.28±0.13	5.27±0.11	0.71	0.89
50-60	147.8±1.5	22.0±4.0	4.38±0.12	4.55±0.10	0.69	0.96
60-70	144.6±1.7	21.6±4.6	3.63±0.11	3.91±0.09	0.68	1.12
70-80	141.8±2.0	24.1±5.7	2.84±0.10	3.22±0.09	0.66	1.23
80-92	140.6±2.5	14.3±7.1	2.24±0.10	2.77±0.09	0.63	1.13

STAR @ $\sqrt{s_{NN}} = 200$ GeV

PRL **92**, 112301 (2004)

Centr. [%]	T [MeV]	μ_B [MeV]	ρ_{max} [fm]	τ [fm]	$\beta_{\perp}^{max}$	χ^2/ NDF
0-5	160.0±1.2	24.0±2.2	9.22±0.31	7.13±0.19	0.79	0.30
5-10	160.6±1.2	25.0±2.2	8.34±0.28	6.75±0.18	0.78	0.27
10-20	161.2±1.1	22.9±2.2	7.45±0.24	6.17±0.16	0.77	0.22
20-30	162.3±1.1	23.1±2.2	6.31±0.20	5.60±0.14	0.75	0.25
30-40	162.0±1.1	20.4±2.2	5.38±0.17	5.15±0.12	0.72	0.19
40-50	163.0±1.1	21.0±2.2	4.46±0.14	4.64±0.11	0.69	0.13
50-60	163.4±1.1	18.8±2.3	3.67±0.12	4.13±0.10	0.66	0.13
60-70	162.4±1.1	16.5±2.3	2.95±0.10	3.79±0.09	0.61	0.26
70-80	163.7±1.2	15.8±2.5	2.22±0.09	3.16±0.08	0.57	0.61

The freeze-out temperature

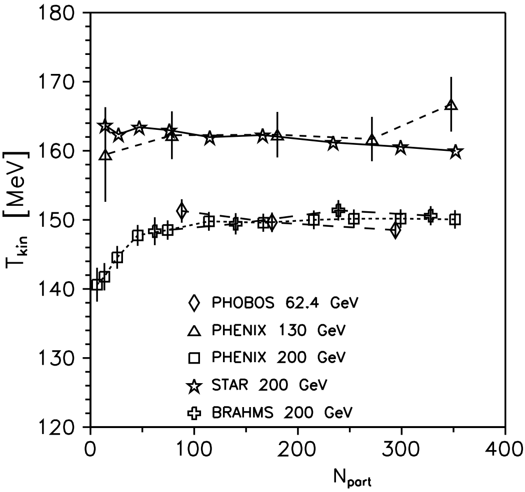

Figure: Centrality dependence of the freeze-out temperature for RHIC measurements at $\sqrt{s_{NN}} = 62.4,\ 130$ and 200 GeV.

The baryon number chemical potential

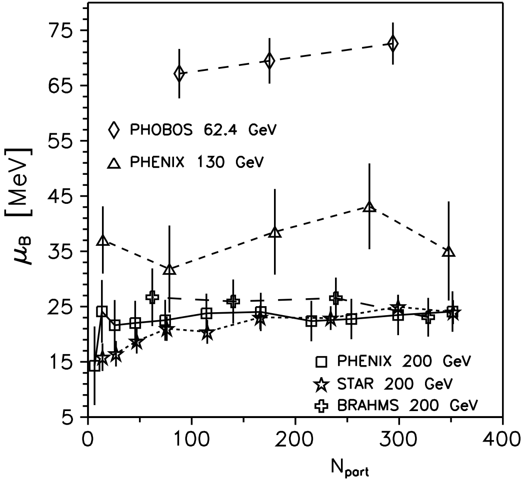

Figure: Centrality dependence of the baryon number chemical potential at the freeze-out for $\sqrt{s_{NN}} = 62.4,\ 130$ and 200 GeV.

Conclusions

- The statistical model well describes yields and spectra of particles produced in heavy-ion collisions.
- The freeze-out temperature and baryon number chemical potential obtained in the model depend weakly on the centrality of the collision.
- For the RHIC range of collision energy the freeze-out temperature is $T_{f.o.} = 150 - 160$ MeV, what is of the order of 10^{12} K !