Self consistent approach to the lepton-nucleus scattering at intermediate energy transfers

Neutrino Seminar 09.01.2012

Jakub Żmuda

jazmuda@ift.uni.wroc.pl

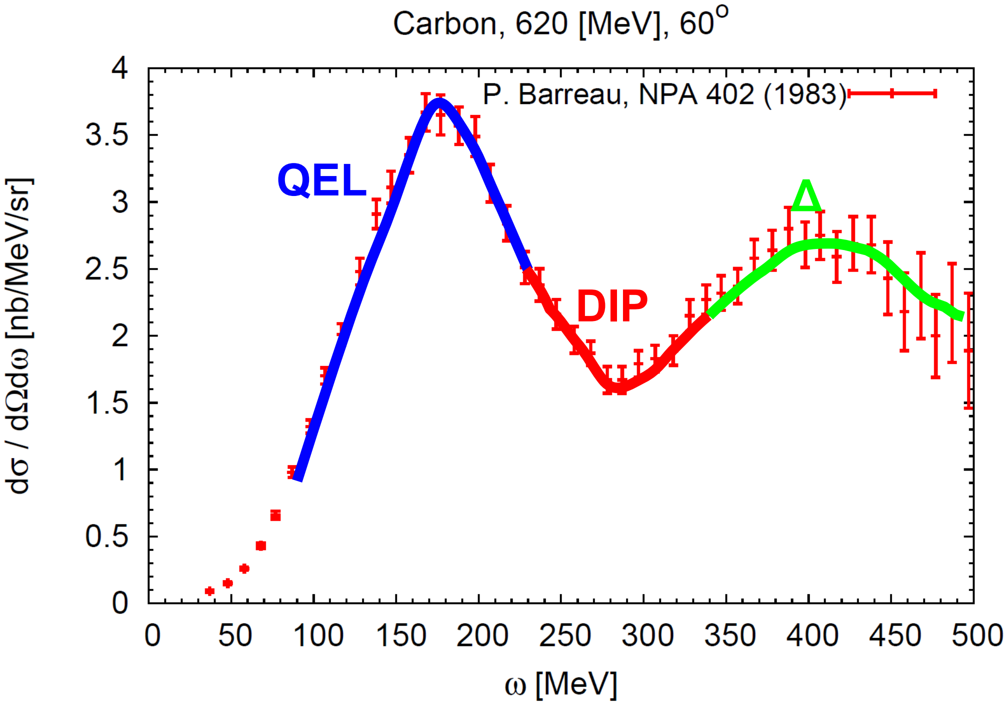
Institute of Theoretical Physics, University of Wroclaw

Advisor: Jan Sobczyk

Motivation - neutrino experiments

- Neutrino beams with energies peaked below 1 [GeV]: modern experiments (MiniBooNe, T2K...)
- Enough energy to reach different types of nuclear dynamics ($1p1h$, $2p2h$, π production)
- The analysis of ν experiments: MC simulations. How do distinguish real $1p1h$ from more complicated cases in a detector?$\rightarrow$ backround subtractions etc.
- Need for an unified microscopic interaction model.
- No new physics without the understanding of ν-nucleus process.
- Electron- the precision probe of nuclear dynamics. Lack of precise neutrino differential cross-sections.

Motivation- nuclear dynamics



- QEL: mainly $1p1h$ excitation, some contribution from $npnh$?
- Δ: mainly excitation of the Δ resonance, (mainly) $1p1h1\pi$ production, but $npnh$ possible!
- DIP: QEL and Δ tails, Meson Exchange Currents, a lot of $2p2h$.

Motivation- what do we demand?

- Energy transfers $\geq\ 50 - 100\ [MeV]$: no real need for the discrete excitations and nuclear resonances
- Demand: in one formalism: $npnh$ excitations, Δ^{1232} resonance, mesons...
- Main focus: model from A. Gil, J. Nieves and E. Oset (NPA 627 (1997) 543-598) and it's recent version from J. Nieves, I. Ruiz Simo, M. J. Vicente Vacas (PRC 83 (2011) 045501).
- The recent extension: relativistic current matrix elements, need for a test against precise electron data.
- From the experimental point of view: what to expect of the dynamics in current MC simulations? What may be missing? How important is $2p2h$ at this energy range?
- Code in C++ both for e and νCC.

General idea of the model

- The inclusive cross-section formula:

$$\frac{d^3\sigma}{d\Omega' dE'} = F_l(Q^2)\frac{|k'|}{|k|}L^l_{\mu\nu}W^{\mu\nu}$$

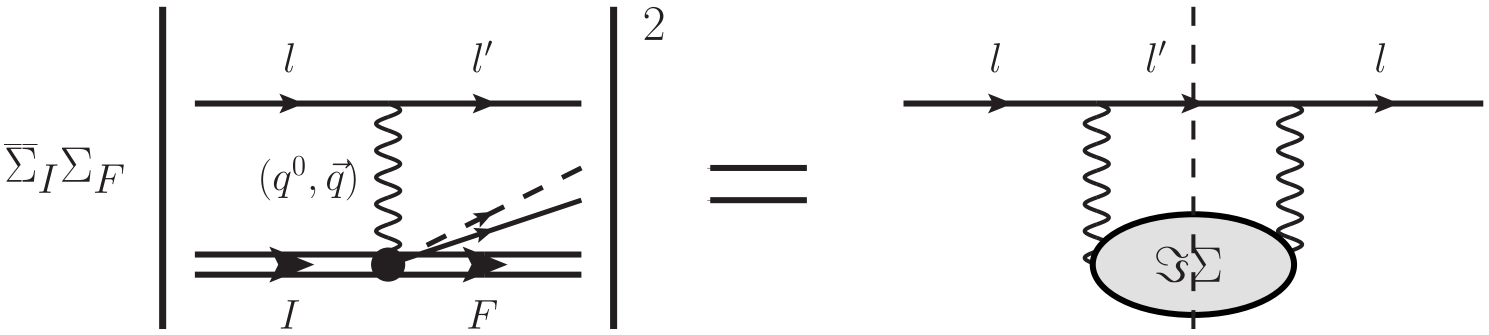
$$F_l(Q^2) = \begin{cases} \frac{2\alpha^2}{Q^4}, \; l=e \\ \frac{G_F^2}{4\pi^2}, \; l=\nu \end{cases}$$

$$L^l_{\mu\nu} = \begin{cases} k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}kk', \; l=e \\ k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}kk' + i\epsilon_{\mu\nu\alpha\beta}k'^\alpha k^\beta, \; l=\nu \end{cases}$$

- Leptons: probe the whole nuclear volume. Nucleus: localized target, rather than infinite Fermi sea.
- LDA: sum (integral) over the responses of proton/neutron Fermi seas, $k_F^N(r) = (3\pi^2\rho^N(r))^{1/3}$.

$$W^{\mu\nu} \to \int d^3r \tilde{W}^{\mu\nu}(r)$$

General idea of the model



- Inclusive cross-section: average over the initial nuclear states and sum over the final ones
- Another point of view: gauge boson self-energy in nuclear medium. Vertical cut: final state particles: lepton l' and hadronic system excitations (nucleon-hole pairs, pions...) on-shell. Imagtinary part: propagator (e.g. $\frac{1}{p^0 - E(p) + i\epsilon}$) in the pole ("Cutkosky rules").

General idea of the model

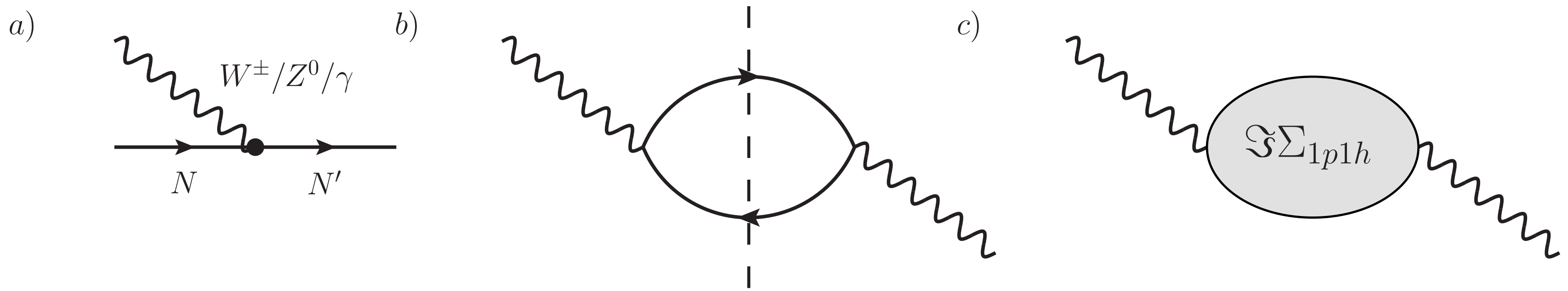
$$\frac{d^3\sigma}{d\Omega' dE'} = F_l(Q^2)\frac{|k'|}{|k|}\int d^3r\left[-\frac{1}{\pi}\Im\left(L^l_{\mu\nu}\Pi^{\mu\nu}(q,\rho(r))\right)\right]$$

$$\Pi^{\mu\nu}(q,\rho(r)) \propto \frac{1}{i}\int d^4x e^{iqx}\left\langle 0(r)\left|T\left\{J^{\nu *}(x)J^{\mu}(0)\right\}\right|0(r)\right\rangle$$

- Medium polarisation tensor $\Pi^{\mu\nu} \to$ gauge boson in-medium self-energy.
- Self-energy $\to$ "black box" with information about the nuclear dynamics.
- Several approaches and approximation to its actual contents: nonrelativistic Many-Body Theory (MBT), Quantum Hadrodynamics (QHD): effective field theory with baryons and mesons,

General idea of the model: $1p1h$

- The most simple example: $1p1h$ Fermi gas model.



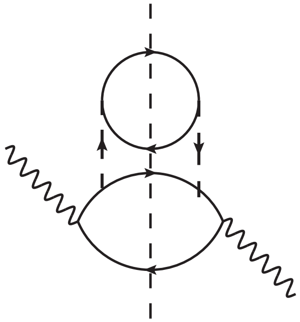
- General prescription:
  1. Take a graph related to one of the transition matrix elements (a)).
  2. Calculate the corresponding buble diagram (b)).
  3. Put the final state particles on the mass shell by the application of Cutkosky rules. You get the imaginary part of the self-energy (c)).

General idea of the model: $1p1h$

- Additional refinements to $1p1h$ in the original paper
- Correction for the experimental energy transfer (Q) values (if the overall nucleus charge changes).

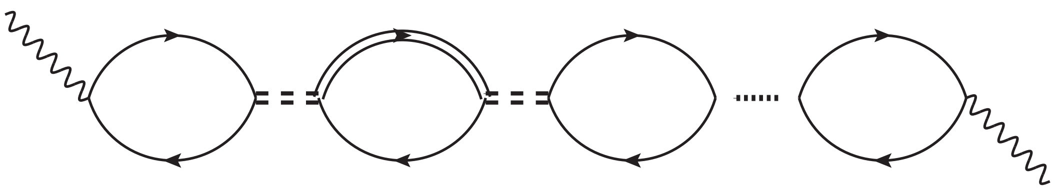
$$\Delta q^0 = {}^{A}_{Z}M - {}^{A\pm1}_{Z}M$$

- My $1p1h$ for electrons: still mean binding energy in use (no charge exchange through γ).
- Possible Spectral Function (SF- dressing of the initial nucleon state, "hole")/Final State Interaction (FSI- dressing of the final nucleon, "particle"), not yet in my code



$1p1h$

- ph+Δh RPA with Landau-Migdal nonrelativistic potentials, but in the most recent version relativistic ph bubbles (RPA not yet in my code)



$$V(\rho) = c_0\left[f_0(\rho) + f_0'(\rho)\boldsymbol{\tau_1\tau_2} + g_0(\rho)\boldsymbol{\sigma_1\sigma_2} + g_0'(\rho)\boldsymbol{\tau_1\tau_2\sigma_1\sigma_2}\right]$$

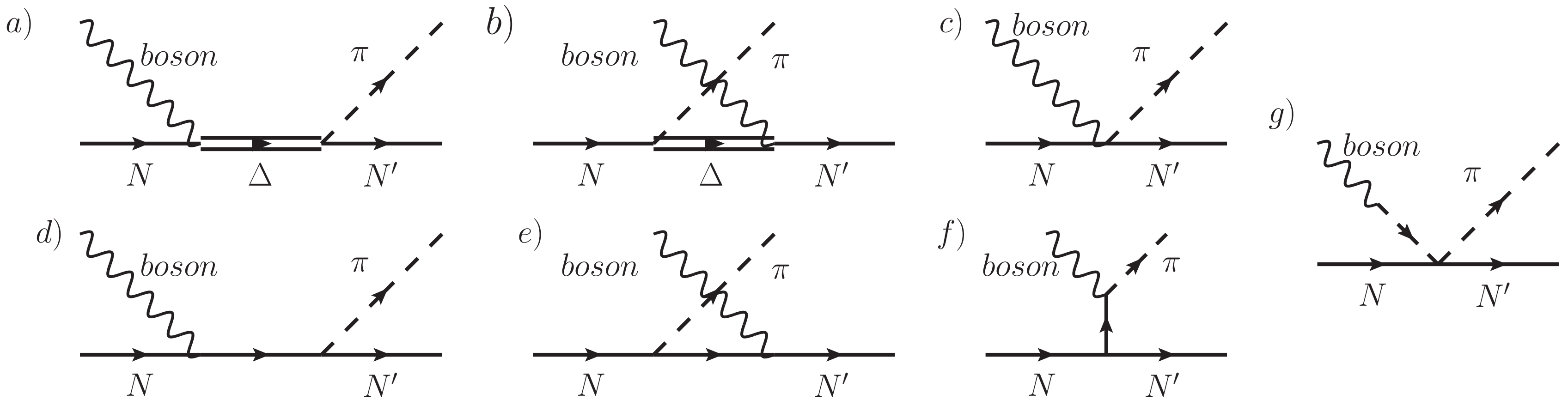
- Explicit $\pi + \rho$ exchanges:

$$c_0 g_0'(\rho)\boldsymbol{\tau_1\tau_2\sigma_1\sigma_2} \rightarrow \boldsymbol{\tau_1\tau_2}\sum_{i,j=1}^{3}\sigma_1^i\sigma_2^j(\hat{q}_i\hat{q}_j V_l(q) + (\delta_{ij} - \hat{q}_i\hat{q}_j)V_t(q))$$

$$V_l(q) = \frac{f_\pi^2}{m_\pi^2}\left(F_\pi(q^2)\frac{\boldsymbol{q}^2}{q^2 - m_\pi^2} + g_l'\right),\ V_t = \frac{f_\pi^2}{m_\pi^2}\left(C_\rho F_\rho(q^2)\frac{\boldsymbol{q}^2}{q^2 - m_\rho^2} + g_t'\right)$$

$1p1h1\pi$ Δ + MEC

- Vertices and currents: nonlinear σ-model with sprontaneous symmetry breaking patterns from QCD + phenomenological form factors + Δ (from E. Hernandez, J. Nieves, M. Vacas PRD 76 (2007) 033005).



- Graphs: a) Delta Pole (DP), b) Delta Pole Crossed (DPC), c) Contact Term (CT), d) Nucleon Pole (NP), e) Nucleon Pole Crossed (NPC), f) Pion Pole (PP). Currents treated in a fully relativistic manner.
- Boson: $W^{\pm}$, γ. f): only $W^{\pm}$

$1p1h1\pi$ Δ + MEC

- Graphs; simple, algebra: not quite. Example: Δ spin-$3/2$ resonance vertex and propagator:

$$\left\langle \Delta^+(p_\Delta = p+q) \left| j^\mu_{CC+}(0) \right| n(p) \right\rangle = \overline{u}_{s_\Delta}(\boldsymbol{p_\Delta})_\alpha \Gamma^{\alpha\mu}(p,q) u_s(\boldsymbol{p})$$

$$\Gamma^{\alpha\mu}(p,q) = \left[\frac{C_3^V}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \frac{C_4^V}{M^2}(g^{\alpha\mu} q\cdot p_\Delta - q^\alpha p^\mu_\Delta) + \right.$$

$$\left. + \frac{C_5^V}{M^2}(g^{\alpha\mu} q\cdot p - q^\alpha p^\mu) + \frac{C_6^V}{M^2} q^\alpha q^\mu \right] \gamma^5 + \left[\frac{C_3^A}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \right.$$

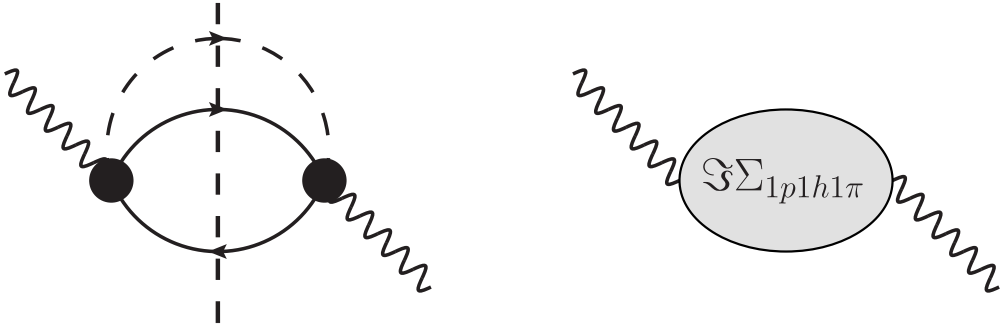
$$\left. + \frac{C_4^A}{M^2}(g^{\alpha\mu} q\cdot p_\Delta - q^\alpha p^\mu_\Delta) + \frac{C_5^A}{M^2} g^{\alpha\mu} + \frac{C_6^A}{M^2} q^\alpha q^\mu \right]$$

$$G^{\mu\nu}(p_\Delta) = \frac{P^{\mu\nu}_{3/2}(p_\Delta)}{p^2_\Delta - M^2_\Delta + iM_\Delta\Gamma_\Delta}$$

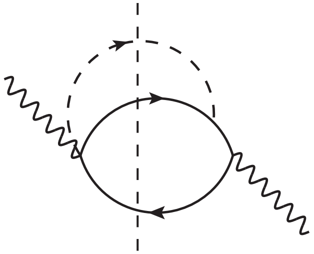
- 2 ways out: Mathematica & copy+paste or numerical treatment.

$1p1h1\pi$ Δ + MEC

- Sum of all channels (a)-f))+ their "interferences" (36-49 graphs):



- Sum of all amplitudes leading to $1p1h1\pi$ final states, "interference":

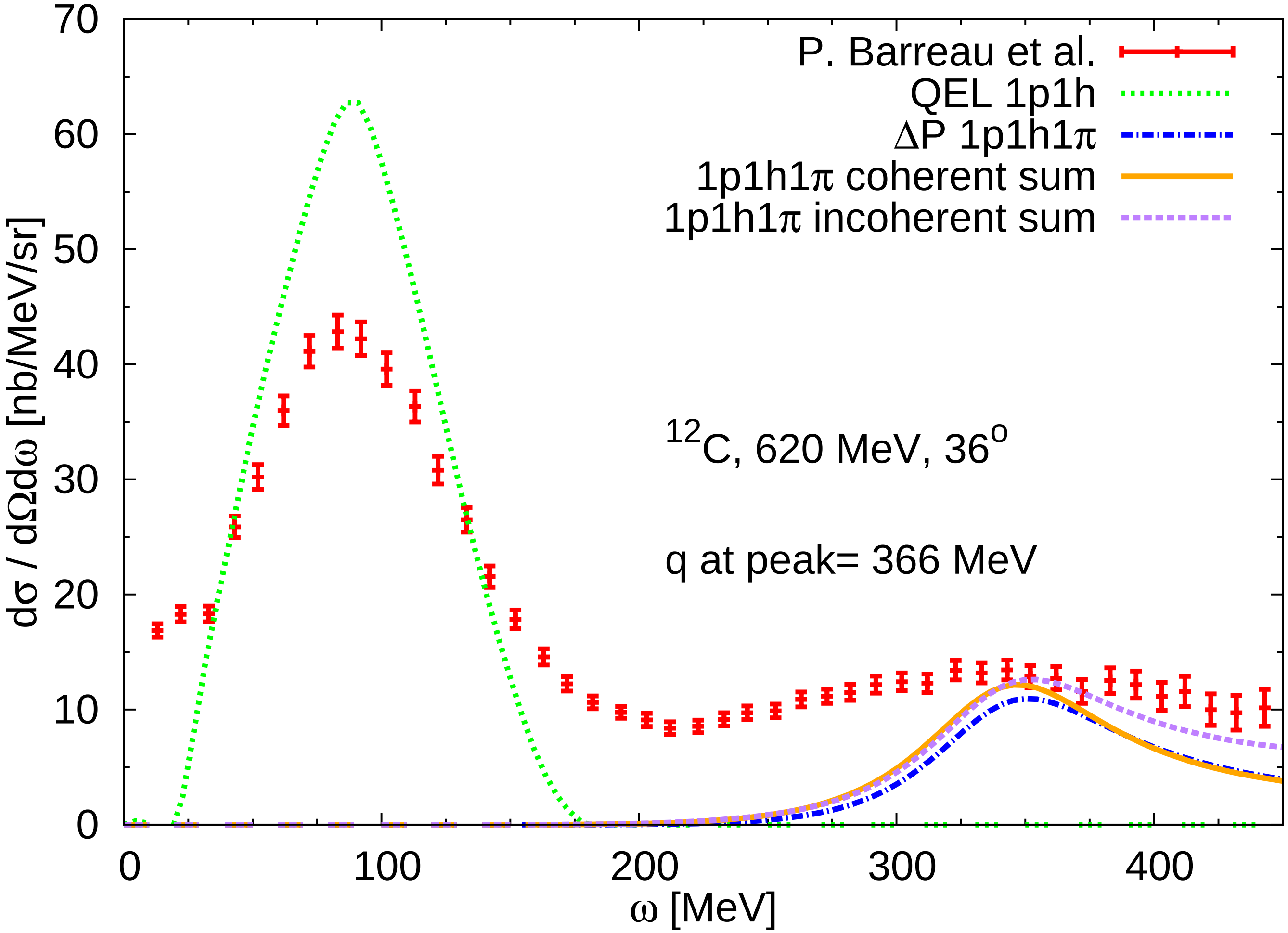


- Major part of the primary 1π-production (resonant+ part of possible nonresonant background)
- Free Δ width still taken at this level...

Preliminary tests

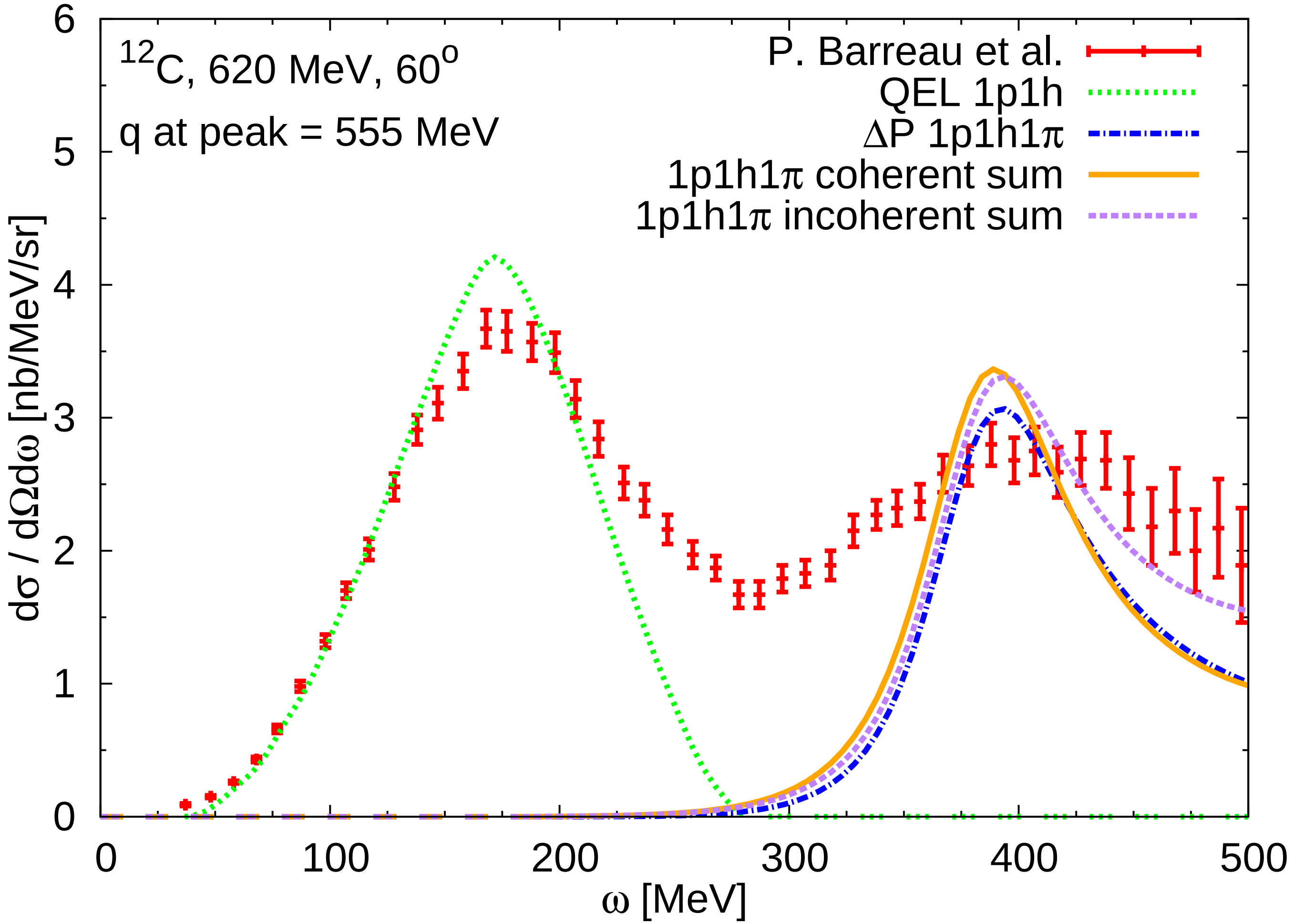
- All data taken from:
  http://faculty.virginia.edu/qes-archive/QES-data.php
- Still no 2p2h, how much one does miss (MC generators)?

Preliminary results $1p1h1\pi$, $^{12}C(e,e')$



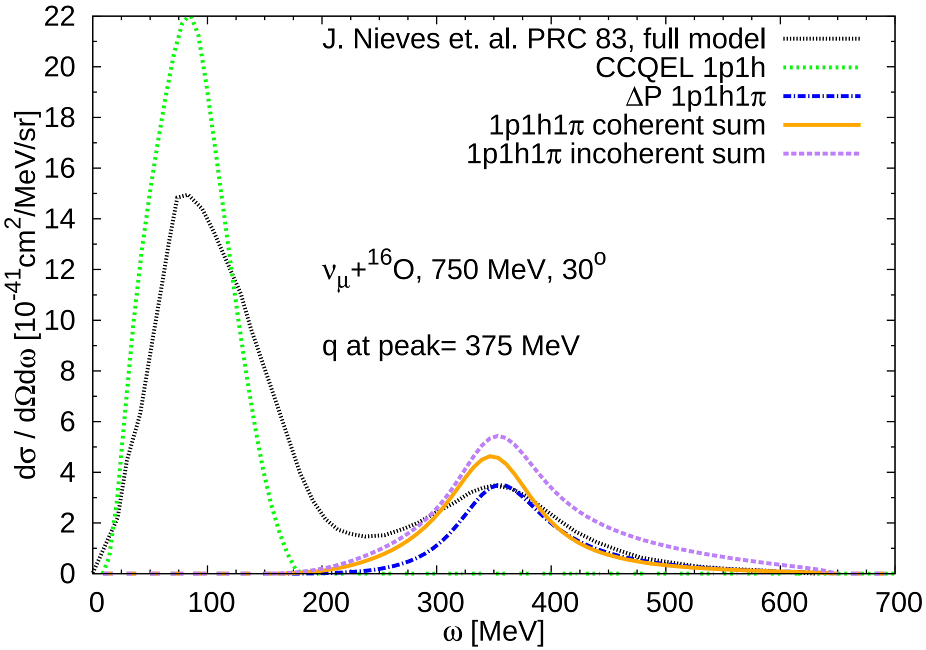
- QEL too high in LFG$\to$ need for RPA at low $|q|$, not enough cross-section in DIP, space for more mechanisms. "Coherent" (amplitudes) and "incoherent" (cross-section) sums different!

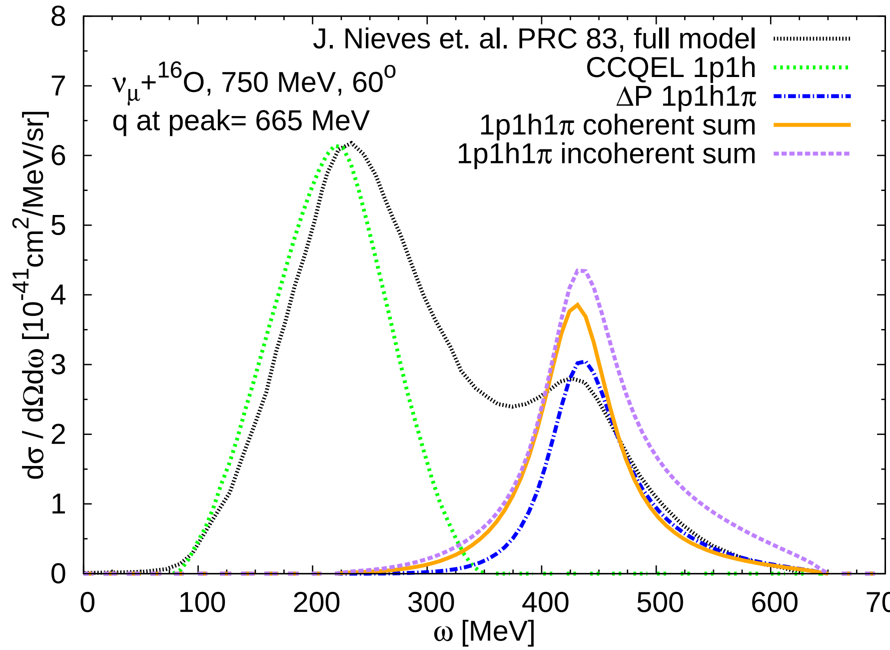
Preliminary results $1p1h1\pi$, $^{12}C(e,e')$



- $1p1h$ RPA less important, free Δ too narrow $\rightarrow$ in-medium broadening by multinucleon absorptions important! DIP- even more important at high angles.

Preliminary results $1p1h1\pi$, $^{16}O + \nu_\mu$

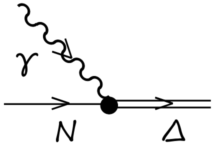




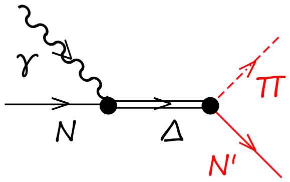
- The same conclusions, as for electrons. ν cross-sections behave almost the same
- Still far from the results of the full model, work in progress!
- Introduction of the nonresonant $1p1h1\pi$ background can not explain size of the DIP cross-section.

The $\Delta - h$ excitation, physics

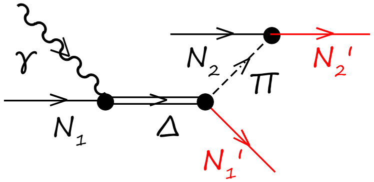
- First: excitation of Δ through $\gamma N \Delta$ vertex :



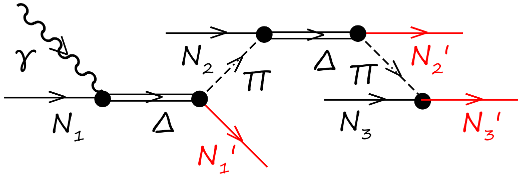
- Δ unstable, decay to pion ($\gamma N(\Delta) \to N' + \pi$):

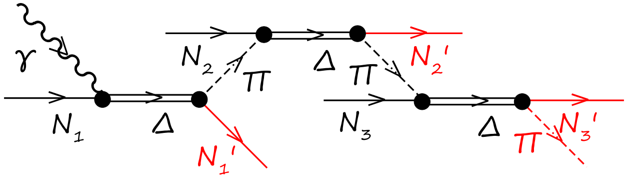


- pion excites another ph pair ($\gamma N(\Delta) \to 2p2h$):



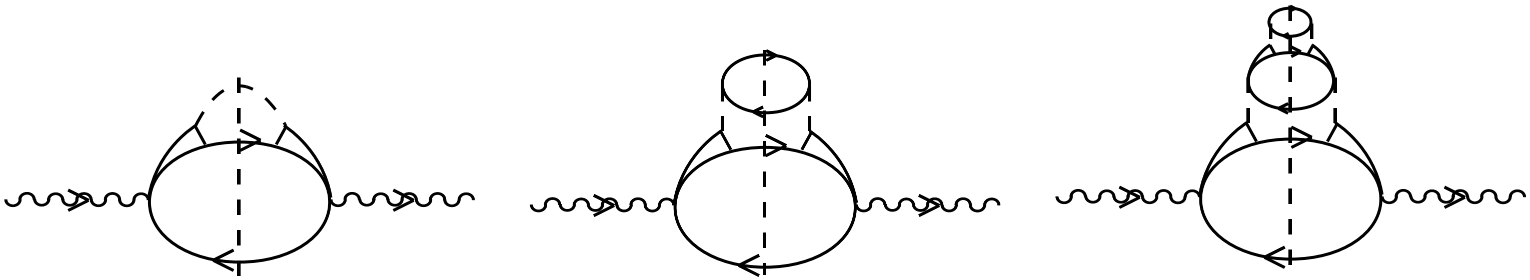
- ... or another Δh ($\gamma N(\Delta) \to (1p1h + \Delta h) \to 3p3h$), ($\gamma N(\Delta) \to (1p1h + \Delta h) \to 2p2h1\pi$):





The $\Delta - h$ excitation, E. Oset, L. L.Salcedo

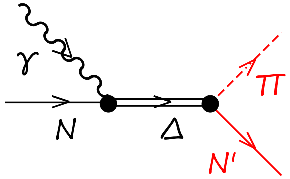
- Different physical channels parametrised in $\Im\Sigma_\Delta$ (NPA 468 (1987) 631-652):
    1. 1π production: $\frac{1}{2}\tilde{\Gamma} - \Im\Sigma_{QEL}$, $\frac{1}{2}\tilde{\Gamma}$: $\Delta \to N\pi$ decay width with the nucleon PB correction, $\Im\Sigma_{QEL} \to$ many-body corrections.
    2. $2p2h$ excitation: $-\Im\Sigma_{A2}$
    3. $3p3h$ excitation: $-\Im\Sigma_{A3}$



- Σ parametrisation: powers of $\frac{\rho(x)}{\rho_0}$ and LDA.

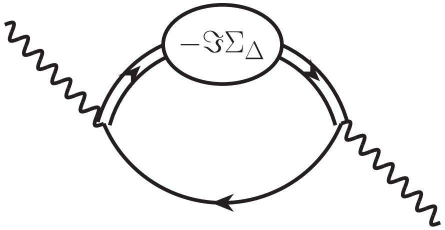
The $\Delta - h$ excitation, E. Oset, L. L. Salcedo

- Some channels explicit in the $1p1h1\pi$ part:



- To do: Avoiding double-counting: subtraction of DP^2 $1p1h1\pi$ from the previous considerations, add $1p1h1\pi$ part separately from the NPA 468 self-energy (full Dyson re-sumation).

- To do: add also $2p2h$ and $3p3h$ parts separately from the NPA 468 self-energy (full Dyson re-sumation).



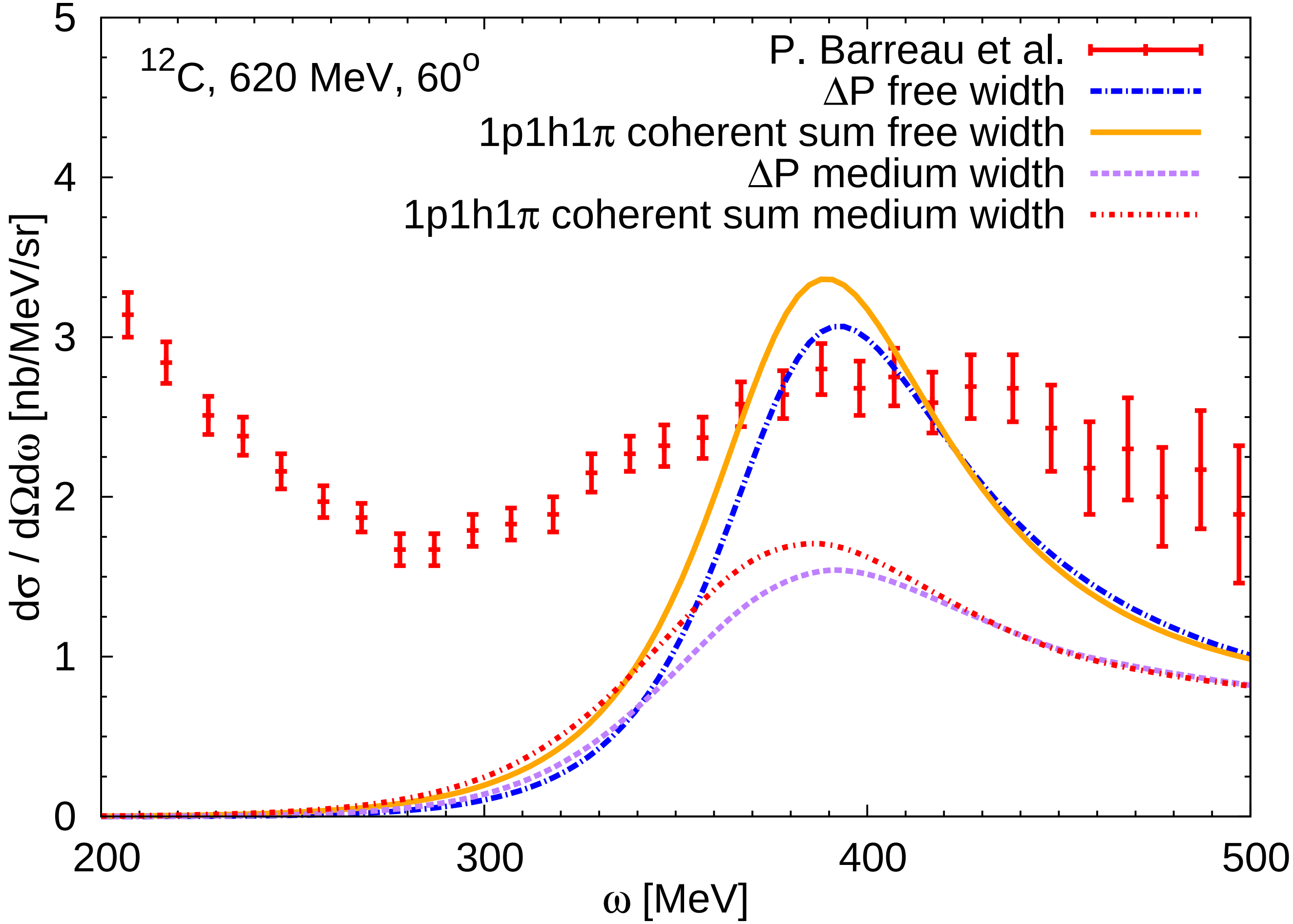
The $\Delta - h$ excitation, E. Oset, L. L. Salcedo

- Already included: full medium modification of the Δ propagator, together with the real part of selfenergy with Δh RPA for the electrons.

$$\Re\Sigma_\Delta \approx (-70 + 0.133 * q^0)[MeV] + \frac{4}{9}\left(\frac{f^*}{m_\pi}\right)^2 \rho V_t$$

- "Little inconsistency": self-energy from all channels up to $3p3h$ in the denominator, but no modification in the numerator, just to show the importance of Δ medium broadening.

Preliminary: Δ in-medium, $^{12}C(e, e')$



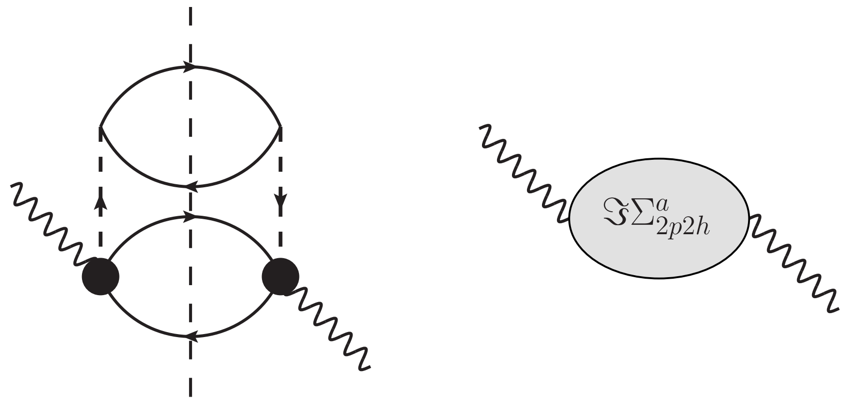
- Only one-loop level $1p1h1\pi$ included in the numerator, no cross-section from $npnh$ included yet!
- Delta self-energy in the denominator of propagator: medium effects are large. A lot of space for the $npnh$ contributions.

More to do

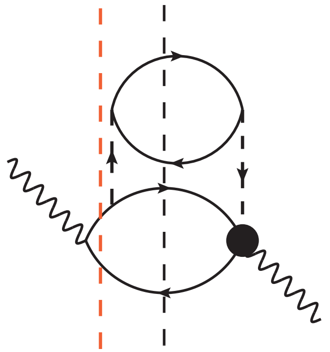
- Quite a lot...

$2p2h$

- The first type of contribution:



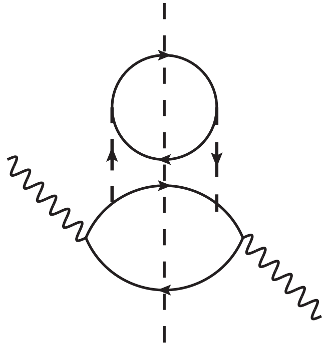
- One more ph bubble on the pion line. Pion now virtual.



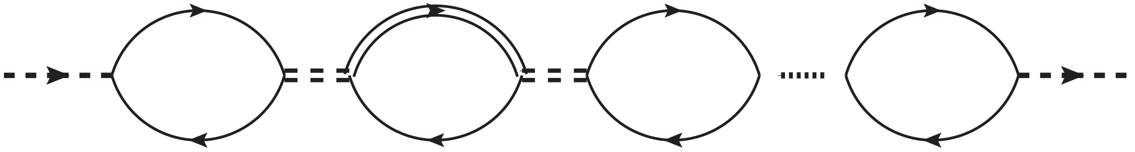
- Special treatment of the graphs containing NP needed. Real singularity in the intermediate nucleon propagator!

$2p2h$

- Even more precaution for the NP^2: sometimes already accounted for in the FSI/particle SF.

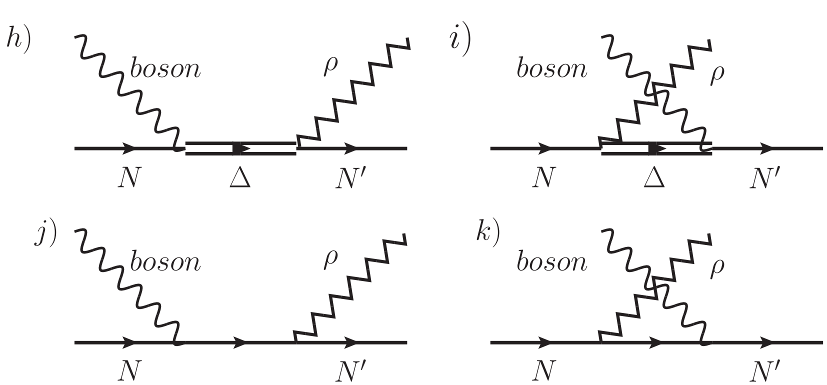


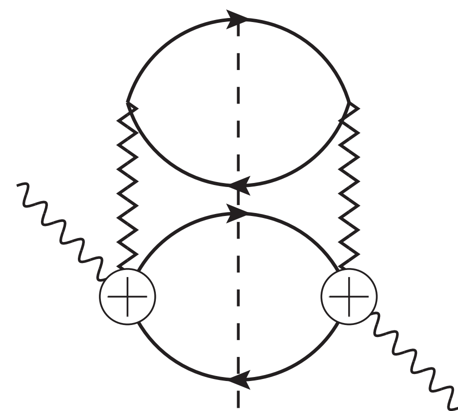
- Refinement in this part: π propagator fully dressed with the RPA:



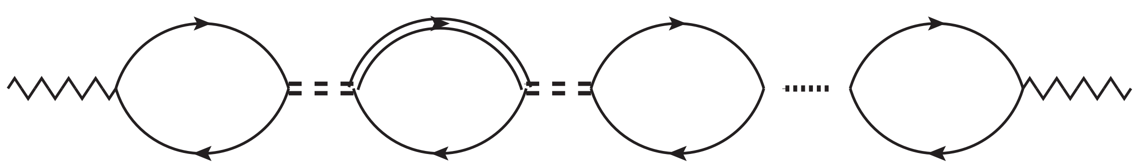
$2p2h$

- Additional ρ-exchange driven interactions in this part of the 2p2h



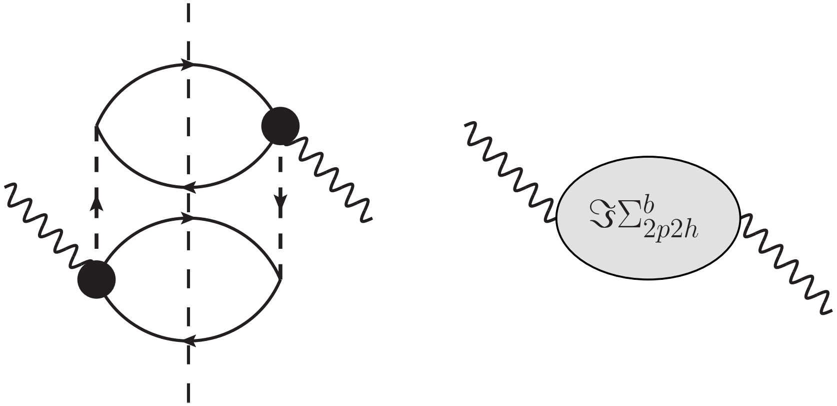


- Exlusion of $h)^2$. Accounted for in the Δ self energy.
- ρ propagator fully dressed with the RPA (interaction different, than for the π, V_t in place of the V_l):

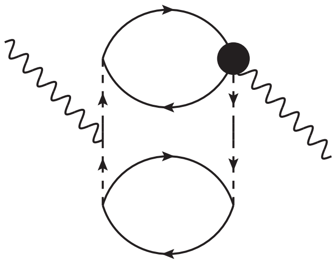


$2p2h$

- Last type of the $2p2h$ contribution:



- Now each boson coupled to a different ph bubble.



- However, these types can be clasified as both $2p2h$ topologies, easy to double-count by a mistake.
- Pion RPA also present here.

Summary

- This type of model- very flexible, allowing for inclusion of different dynamics in a self-consistent way. One needs basically an appropriate Lagrangian, form-factor sets and experimental density profile to perform quite advanced many-body calculations.
- Medium modifications of the Δ propagator give large effects on the cross-section.
- The nonresonant backroung for $1p1h1\pi$ channel should be added on the amplitude levels, i.e. using the interferences between all possible mechanisms. It does not cover the lack in cross-section in the DIP region.
- Plans for the nearest future: implementation of the full model starting with $2p2h$, extensive tests for electrons. How important are the multinucleon channels?
- Are the presented mechanisms enough to cover the whole cross-section from QEL to Δ peaks, filling the DIP?

Thank you!

- This work was sponsored from the European Social Fund.

Bibliography

- **The model of inclusive (e, e') process:** A. Gil, J. Nieves, E. Oset, Nucl. Phys. **A627** (1997) 543-598.
- **Neutrinos, recent:** J. Nieves, I. Ruiz Simo, M. J. Vicente Vacas, Phys.Rev. C83 (2011) 045501
- **Pion production model:** E. Hernandez, J. Nieves, M. Valverde, Phys.Rev. D76 (2007) 033005
- **Δ self-energy:** E. Oset, L. L. Salcedo, Nucl. Phys. **A468** (1987) 631-652.
- **LDA parameters:** C. W. De Jager, H. De Vries, C. De Vries, Atom. Data Nucl. Data Tabl. **14** (1974) 479-508.
  H. De Vries, C. W. De Jager, C. De Vries, Atom. Data Nucl. Data Tabl. **36** (1987) 495-536.
- **Some of the other important papers in the subject:**
- J. W. Van Orden, T. W. Donnelly, Annals Phys. **131** (1981) 451-493.
- J. E. Amaro, C. Maieron, M. B. Barbaro *et al.*, Phys. Rev. **C82** (2010) 044601. [arXiv:1008.0753 [nucl-th]].
- E. Oset, L. L. Salcedo, D. Strottman, Phys. Lett. **B165** (1985) 13-18.
- J. Marteau, Eur. Phys. J. **A5** (1999) 183-190. [hep-ph/9902210].
- M. Martini, M. Ericson, G. Chanfray *et al.*, Phys. Rev. **C80** (2009) 065501. [arXiv:0910.2622 [nucl-th]].