Diffractive scattering of hadrons through nuclei

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Wroclaw, June 15, 2009

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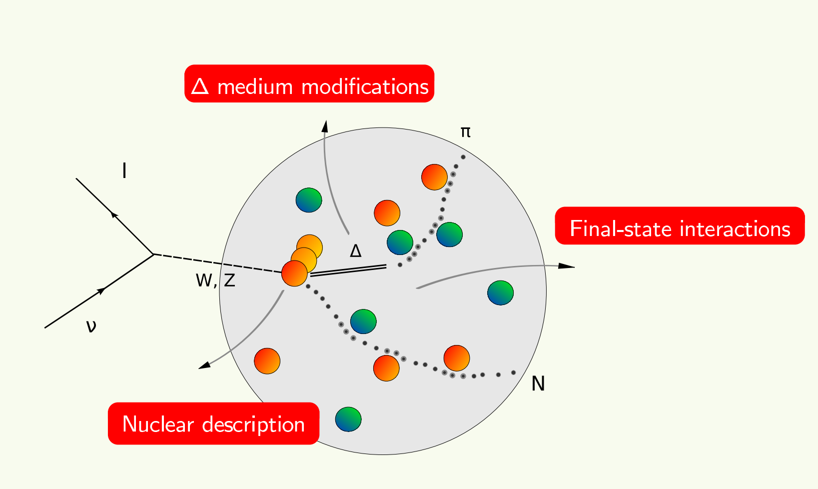
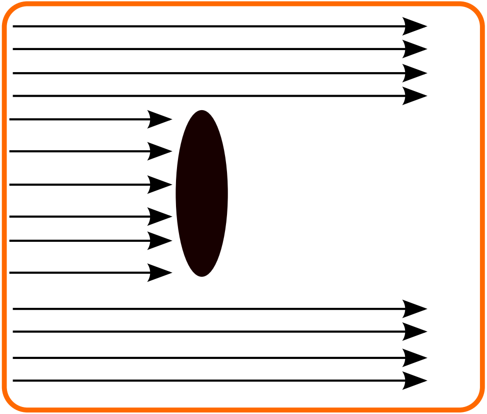
The trouble with nuclear reactions ...



Outline

- Relativistic formulation of Glauber multiple-scattering theory
- How to implement short-range correlations in Glauber calculations?
- Nuclear transparencies extracted from ^{4}He$(\gamma, p\pi^-)$ and $A(e, e'\pi^+)$
- Robustness of the eikonal results for the nuclear transparencies
  - Comparison with semi-classical calculations
  - Consistency with transparencies extracted from $A(e, e'p)$ and $A(p, 2p)$
  - Second-order eikonal corrections
- Conclusions

Let's do some optics



Black Disk scattering

- $\phi_{out} = \phi_{in} + \phi_{scatt.}$
- $\phi_{scatt.} = -\phi_{in}$ in area behind disk
- When $kR \gg 1$, the cross section is strongly forward peaked: Fraunhofer diffraction
- Grey disk scattering $\rightarrow$ introduction of a Profile function $\Gamma(\vec{b})$ with a Gaussian form $\phi_{scatt.} = -\Gamma(\vec{b})\phi_{in}$

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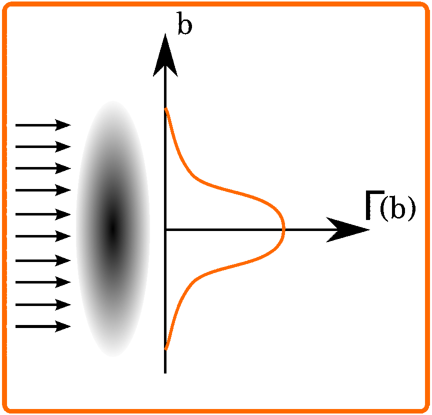

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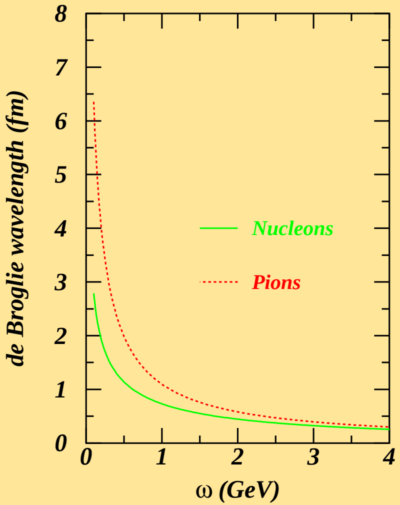
Glauber multiple-scattering theory

- Multiple-scattering theory for the passage of an energetic particle (λ) through a medium with range R valid when

$$\lambda < r_S < R$$

  r_S : the interaction range between the particle and the objects in the medium
- First-order eikonal method ; adopts the frozen approximation for the scattering centers and the mean-field approximation!
- Relativistic Multiple-Scattering Glauber approximation (RMSGA) NPA **A728** (2003) 226
- The RMSGA provides a common theoretical framework for computing cross sections for
  1. exclusive reactions like $A(e,e'p)$, $A(e,e'pp)$, $A(p,2p)$, $A(e,e'p\pi^-)$
  2. quasi-elastic contributions to inclusive responses like $A(\nu,\nu')$

What is the applicable energy range?



- Momentum of the residual nucleus can be neglected relative to its rest mass

$$\lambda = \frac{1}{p_{N(\pi)}} = \frac{1}{\sqrt{2\omega M_{N(\pi)} + \omega^2}} .$$

- πN and $N'N$ interaction ranges are of the order of fm.
- Eikonal approximation can be used down to nucleon kinetic energies of $\approx$300 MeV. Corresponds to pion energies of about 750 MeV.

Relativistic Multiple-Scattering Glauber Approximation

- Model adopts the mean-field approximation with bound-state wave functions from the $\sigma - \omega$ model (Serot-Walecka).
- Intranuclear attenuation on the impinging or escaping hadron i is implemented by means of a

DIRAC-GLAUBER PHASE OPERATOR $\mathcal{G}(\vec{b}, z)$ (SCALAR)

$$\widehat{\mathcal{G}}\left(\vec{r}, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_A\right) \equiv \prod_{j=2}^{A} \left[1 - \Gamma_{iN}\left(\vec{b} - \vec{b}_j\right) \theta\left(z - z_j\right) \right]$$

Product extends over all spectator nucleons!

- Profile function reflects diffractive nature of πN and $N'N$

$$\Gamma_{iN}(\vec{b}) = \frac{\sigma_{iN}^{tot}(1 - i\epsilon_{iN})}{4\pi\beta_{iN}^2} \exp - \frac{\vec{b}^2}{2\beta_{iN}^2} \quad (\text{with}, i = \pi \text{ or}, N') .$$

σ_{iN}^{tot} (total cross section), β_{iN} (slope parameter) and ϵ_{iN} (ratio of real to imaginary part of the amplitude). Obtained from $N'N \longrightarrow N'N$ and $\pi N \longrightarrow \pi N$ data.

Profile Function for Elastic πN scattering

- $\sigma_{\pi N}^{tot}$, $\epsilon_{\pi N}$ and $\beta_{\pi N}$ depend on ejectile's momentum: fits to πN scattering data (PDG and SAID)
- The slope parameter provides a consistency check!

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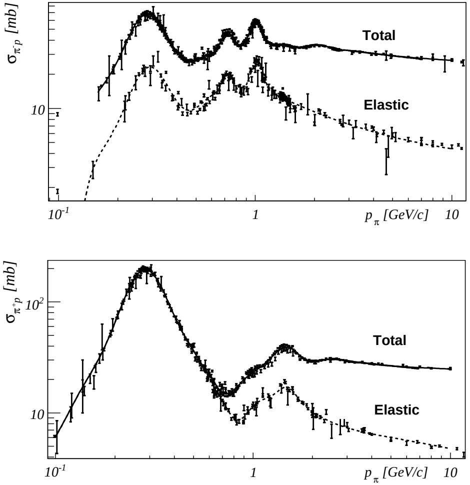

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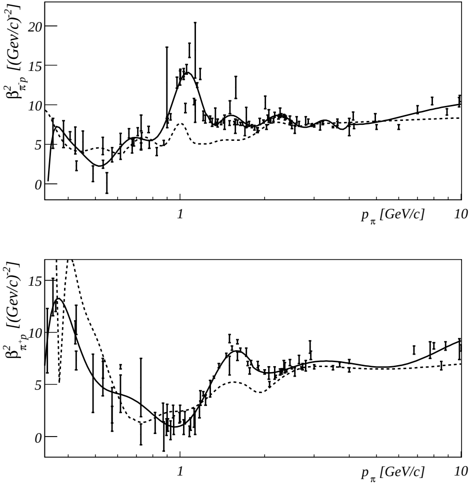

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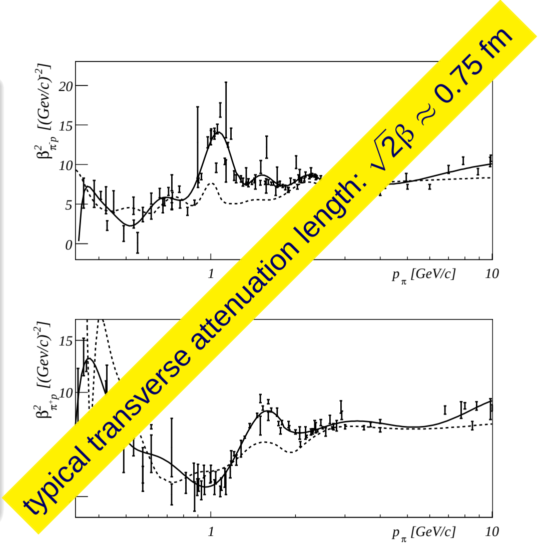


The Dirac Glauber Phase

$$\mathcal{G}(\vec{b},z) = \prod_{\alpha_{occc} \neq \alpha} \left[1 - \int d\vec{r}\,' \left| \phi_{\alpha_{occ}} (\vec{r}\,') \right|^2 \left[\theta\left(z' - z \right) \Gamma \left(\vec{b}\,' - \vec{b} \right) \right] \right] .$$

The Dirac plane wave for an escaping proton/pion gets modulated by (z' along the asymptotic direction of the ejectile)

$$\mathcal{G}(\quad \vec{b} \quad ,z) = 1 - \prod_{\alpha_{occ} \neq \alpha} \left\{ \frac{\sigma_{pN}^{tot} (1 - i\epsilon_{pN})}{4\pi\beta_{pN}^2} \int_0^{\infty} b' db' \int_{-\infty}^{+\infty} dz' \theta(z'-z) \right.$$
$$\left(\left[\frac{G_{n\kappa}(r'(b',z'))}{r'(b',z')} \mathcal{Y}_{\kappa m}(\Omega', \sigma) \right]^2 + \left[\frac{F_{n\kappa}(r'(b',z'))}{r'(b',z')} \mathcal{Y}_{\kappa m}(\Omega', \sigma) \right]^2 \right)$$
$$\left. \times \exp \left[- \frac{(b-b')^2}{2\beta_{pN}^2} \right] \int_0^{2\pi} d\phi_{b'} \exp \left[\frac{-bb'}{\beta_{pN}^2} 2\sin^2 \left(\frac{\phi_b - \phi_{b'}}{2} \right) \right] \right\} .$$

Each target nucleon (scattering center) represented by its own relativistic wave function (upper and lower component)!

Implementing SRC in Glauber calculations (I)

- The independent-particle picture is essential when deriving the Dirac Glauber phase operator

$$\mathcal{G}(\vec{b},z) = \prod_{\alpha_{occ}\neq\alpha} \left[1 - \int d\vec{r}\,' \left|\phi_{\alpha_{occ}}\left(\vec{r}\,'\right)\right|^2 \left[\theta\left(z' - z\right)\Gamma_{pN}\left(\vec{b}\,' - \vec{b}\right)\right]\right] .$$

- The computational cost of the calculations can be considerably (10^3!) reduced by making the following assumption:

$$\left|\phi_{\alpha_{occ}}\left(\vec{r}\,'\right)\right|^2 \longrightarrow \frac{\rho^{[1]}_{A-1}\left(\vec{r}\,'\right)}{A-1}$$

and assuming that $\rho^{[1]}_{A-1}\left(\vec{r}\,'\right)$ are slowly varying functions of $\vec{b}\,'$. Then,

$$\mathcal{G}(\vec{b},z) \approx \exp - \frac{\sigma^{tot}_{pN}\left(1-\epsilon_{pN}\right)}{2}\int_{z}^{+\infty} dz'\rho^{[1]}_{A-1}\left(\vec{b}\,',z'\right)$$

TURNS OUT TO BE A GOOD APPROXIMATION

Implementing SRC in Glauber calculations (II)

- In standard Glauber: effect of intranuclear attenuations is computed as if the density remains unaffected by the presence of a nucleon at $\vec{r} = (\vec{b}, z)$
- One can correct for this bij making the following substitution

$$\rho^{[1]}_{A-1}\left(\vec{b}', z'\right) \rightarrow \frac{A-1}{A-2}\,\frac{\rho^{[2]}_{A-1}\left(\vec{r}', \vec{r}\right)}{\rho_{A-1}\left(\vec{r}\right)}$$

  Conditional one-body density: the density of the residual $A-1$ nucleus given that there is already a nucleon at position $\vec{r}$.
- transverse attenuation length for pions (and nucleons) is of the order of 0.75 fm: **attenuations will be mainly affected by the short-range structure of the transverse density in the residual nucleus**

Implementing SRC in Glauber calculations (III)

- The two-body density can be corrected for the presence of SRC by means of a central correlation function $g\left(\vec{r},\vec{r}'\right)$!!

$$\rho^{[2]}_{A-1}\left(\vec{r}',\vec{r}\right) = \frac{A-2}{A-1}\gamma\left(\vec{r}\right)\rho^{[1]}_{A-1}\left(\vec{r}\right)\gamma\left(\vec{r}'\right)\rho^{[1]}_{A-1}\left(\vec{r}'\right) g\left(\vec{r},\vec{r}'\right)$$

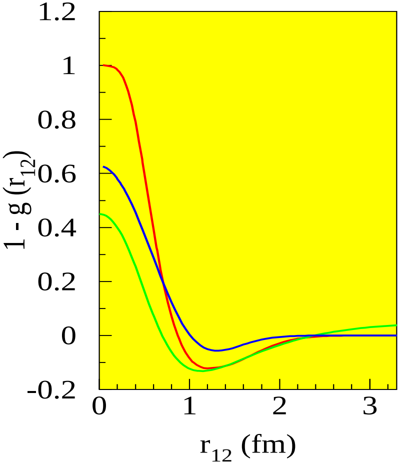
- The γ-functions are introduced to impose the correct normalisation and obey the following integral equation

$$\gamma\left(\vec{r}_1\right)\int d\vec{r}_2 \rho_A\left(\vec{r}_2\right) g\left(\vec{r}_1,\vec{r}_2\right)\gamma\left(\vec{r}_2\right) = A.$$

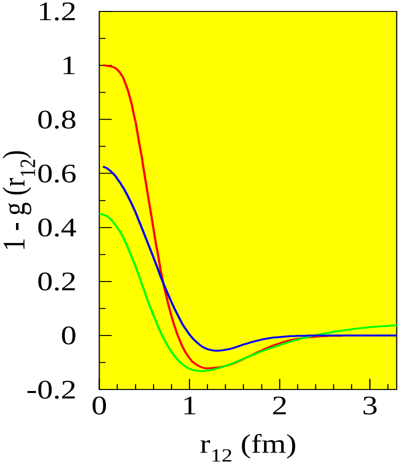
- The introduction of the γ-functions is a very efficient alternative for cluster-expansion methods!

W.Cosyn, M.C. Martínez, J.R., Phys. Rev. C77 (2008) 034602

Implementing SRC in Glauber calculations (IV)



- Choice for the central correlation function $g(r)$?
- The central correlation function has a universal character!
- $^{16}O(e, e'pp)$ and $^{12}C(e, e'pp)$ at MAMI and NIKHEF have provided constraints on $g(r)$!!
- The nuclear $g(r)$ looks like the correlation function for a classical liquid! ((nucleus as a Van der Waals liquid))



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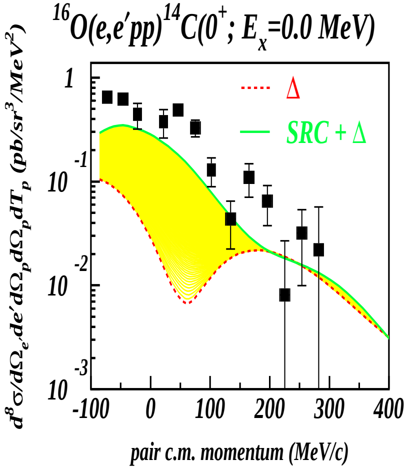
Measurements: MAMI
Theory: Ghent DWIA



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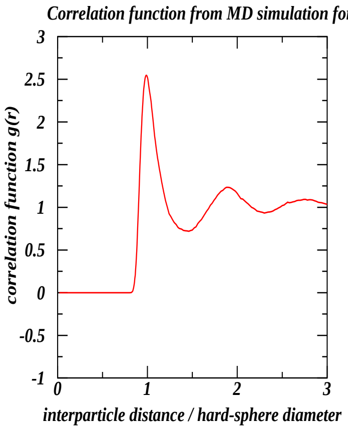
Typical correlation function in a Ar liquid



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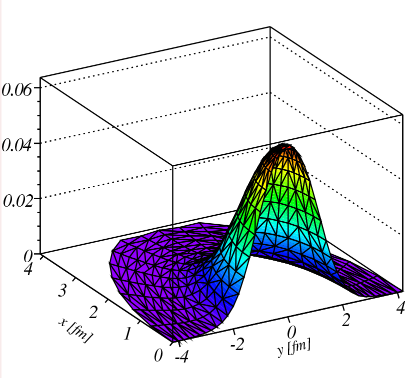
Densities in Glauber calculations (^{4}He case)

A nucleon or pion is created in the center of ^{4}He: how does the nuclear density looks like for this hadron?

^{4}He in IPM



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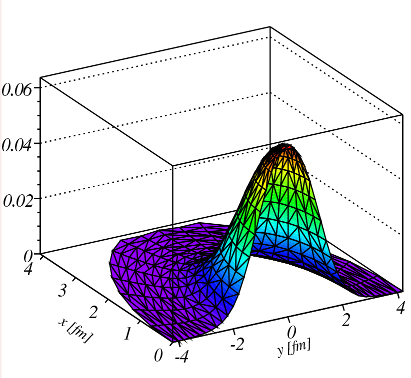

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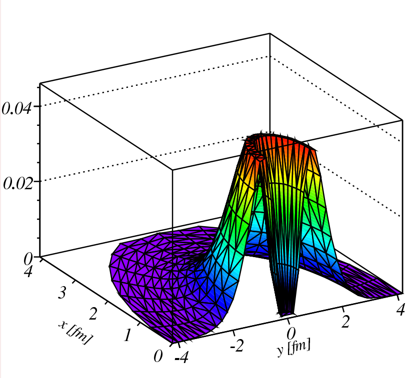
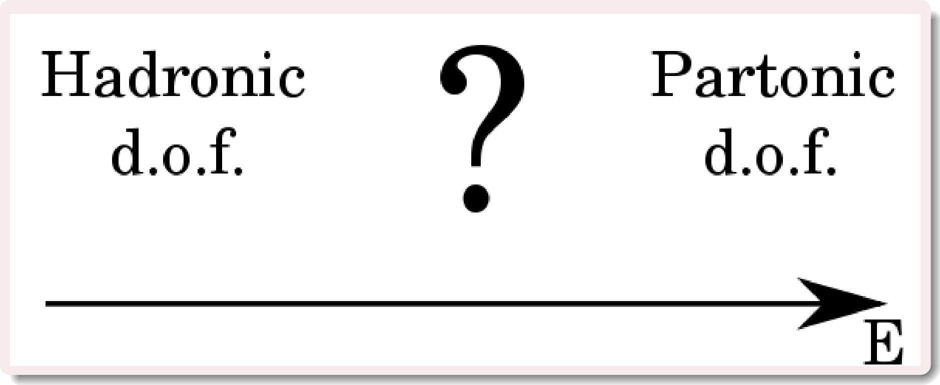


Exploring the crossover

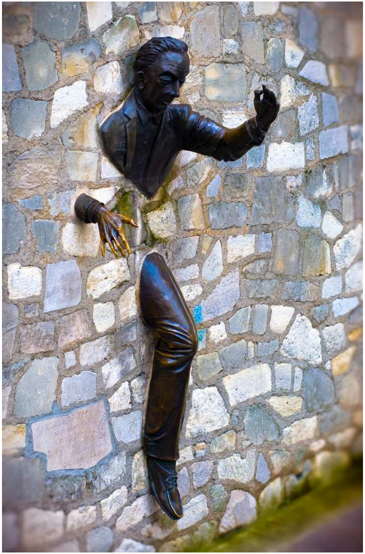


When and how does it occur?

Exploring the crossover

- Look for phenomena predicted in QCD that introduce deviations from traditional nuclear physics observations
- One of these phenomena is color transparency

Color transparency (CT)



- QCD predicts the formations of small-sized hadrons (PLC) in reactions with a high energy transfer Q. The struck quark can only interact with quarks within a distance $\sim 1/Q$ before hadronization occurs.
- The small-sized hadron appears colorless to the medium and hence experiences reduced interactions.
- The PLC will evolve to the normal hadron state as the small-sized configuration is not an eigenstate of the Hamiltonian.

Motivation (I)

- emergence of the concepts of “nuclear physics” (`baryons and mesons`) out of QCD (`quarks and gluons`) remains elusive
- the nuclear transparency as a function of a tunable scale parameter (t or Q^2) is a good quantity to study the crossover between the two regimes
  - one cannot exclude that novel structures of hadronic matter emerge!
  - crossover is a necessary condition for factorization to apply (`extraction of GPDs from data`)

Nuclear transparency:
effect of nuclear attenuations on escaping hadrons

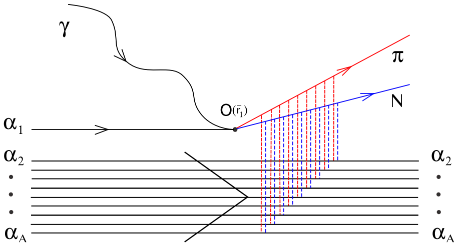
$$T(A, Q^2) = \frac{\text{cross section on a target nucleus}}{A \times \text{cross section on a free nucleon}}$$

Motivation (II)

- interpretation of the transparency experiments requires the availability of reliable and advanced traditional nuclear-physics calculations to compare the data with
- deviations between those model calculations and the measurements point towards the onset of QCD phenomena (`like color transparency`)

Calculating attenuations in $A(\gamma, p\pi^-)$

Separate the cross section in a part describing the fundamental pion production process and a part describing the final state interactions of the pion and proton



Approximations

- Pion production operator in the impulse approximation
- Neglect negative energy contributions of the bound nucleon

Distorted momentum distribution $\rho_D(\vec{p}_m)$

$$\left(\frac{d\sigma}{dE_\pi d\Omega_\pi d\Omega_N}\right)_D \approx \frac{M_{A-1} p_\pi p_N \left(s - (m_N^*)^2\right)^2}{4\pi m_N^* q M_A} f_{rec}^{-1} \rho_D(\vec{p}_m) \frac{d\sigma^{\gamma\pi}}{d\,|\,t\,|}$$

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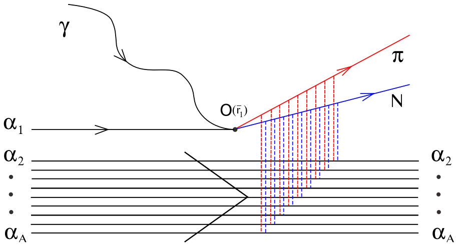

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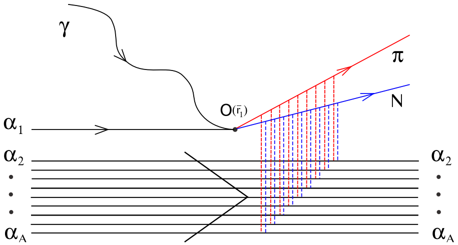

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Color Transparency (I)

- related to the quantum mechanical evolution of wave packets (the small-size configurations which are selected by the probe are no stationary states of the QCD Hamiltonian and evolve with time)
- during expansion: wave packet is subject to reduced attenuations
- **Quantum diffusion parameterization**

$$\sigma_{iN}^{\mathrm{eff}}(z) = \sigma_{iN}^{\mathrm{tot}} \left\{ \left[\frac{z}{l_h} + \frac{< n^2 k_t^2 >}{\mathcal{H}} \left(1 - \frac{z}{l_h}\right) \theta(l_h - z) \right] + \theta(z - l_h) \right\} \quad i = \pi \quad \text{or,}$$

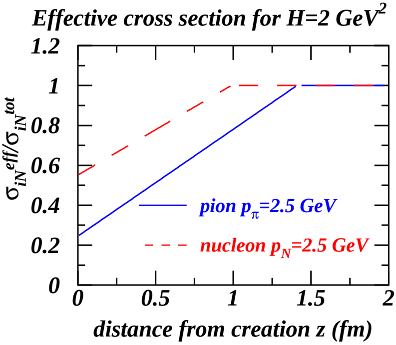
  - the hadronic formation length l_h is related to the mass separation between the different hadronic states and can be estimated from Regge trajectories

$$l_h \ [\mathrm{fm}] \ = \frac{2p_h}{(\Delta m)^2} \approx 0.5 p_h \ [\mathrm{GeV}]$$

  - $k_t \approx 0.35$ GeV educated estimate for the average transverse momentum of a constituent quark in a hadron

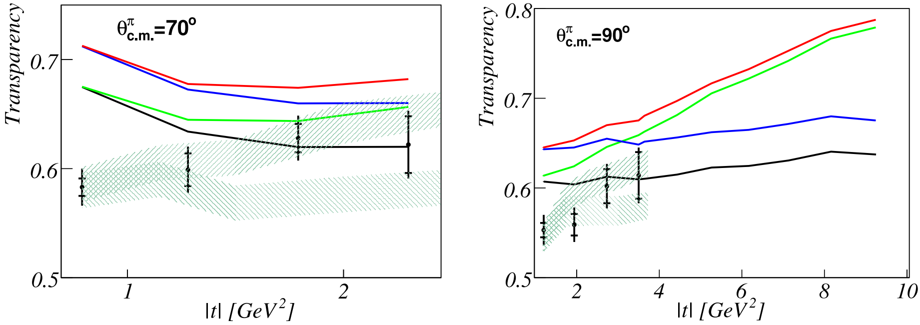
Color Transparency (II)

For a given p_h and hard scale parameter $\mathcal{H}$:
Pion cross section is more strongly reduced and formation length is longer



both the SRC and CT will affect the "effective" density at short transverse distances - can one discriminate between these effects?

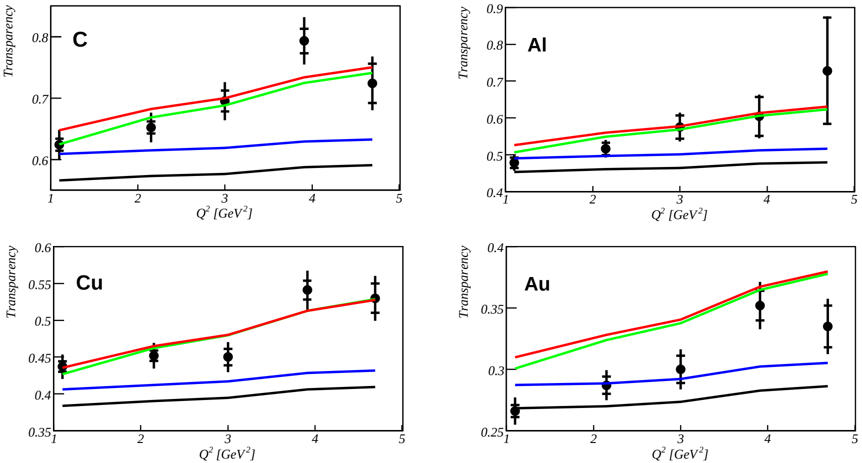
^{4}He$(\gamma, p\pi^-)$ transparencies



Glauber Glauber + SRC Glauber + CT Glauber + SRC + CT

THEORY: W. Cosyn et al., PRC74 (2006) 062201
DATA: D. Dutta et al., PRC68 (2003) 021001
SEMICLASSICAL THEORY: H. Gao et al., PRC54 (1996) 2779

$A(e,e'\pi^+)$ transparencies: Q^2 dependence



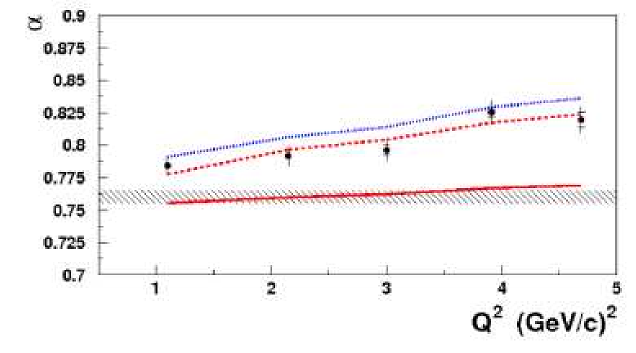
$A(e,e'\pi^+)$ data from JLab ((B. Clasie *et al.*, PRL99 (2007) 242502)

Glauber Glauber + SRC Glauber + CT Glauber + SRC + CT

Robustness of Glauber calculations

- comparison with theoretical predictions rather essential for the interpretation of transparency measurements
- how robust are these theoretical predictions?
  - comparison with other theories?
  - consistent analysis of transparencies extracted from various reactions ($A(e,e'p)$ and $A(p,2p)$)
  - role of higher-order eikonal corrections? ($A(e,e'p)$ as a test case)

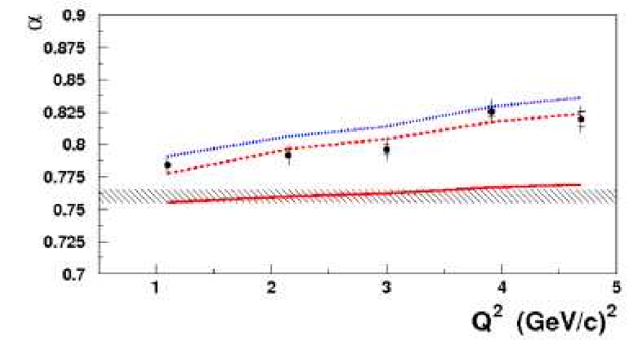
$A(e,e'\pi^+)$ transparencies: A dependence



Fit to the computed (measured) $A(e,e'\pi^+)$ transparencies with $T(A,Q^2) = A^{\alpha(Q^2)-1}$

- hatched band: extracted from πA data
- Red lines: semiclassical Glauber calculations of Larson, Miller, Strikman (PRC74 (2006) 018201) (dashed line includes CT)
- Blue dotted line: RMSGA with CT and SRC

$A(e, e'\pi^+)$ transparencies: A dependence



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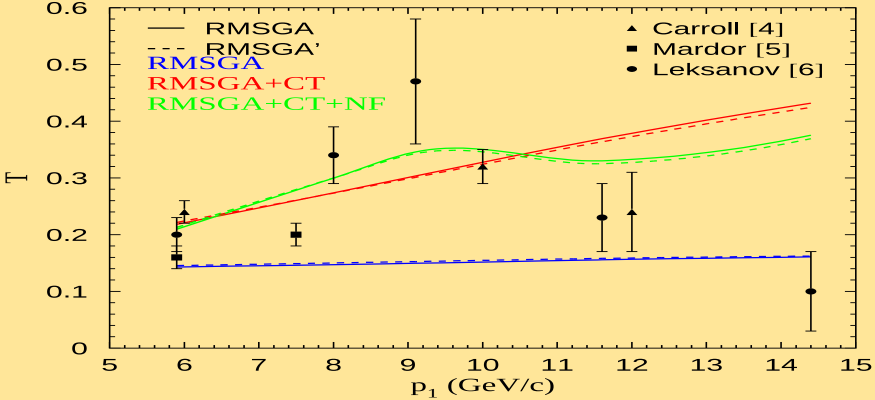
The RMSGA and semi-classical transparencies are similar!!

The nuclear transparency from $^{12}C(p,2p)$ (PLBB644 (2007) 304)



Parameterization of the CT effects compatible with pion production results!

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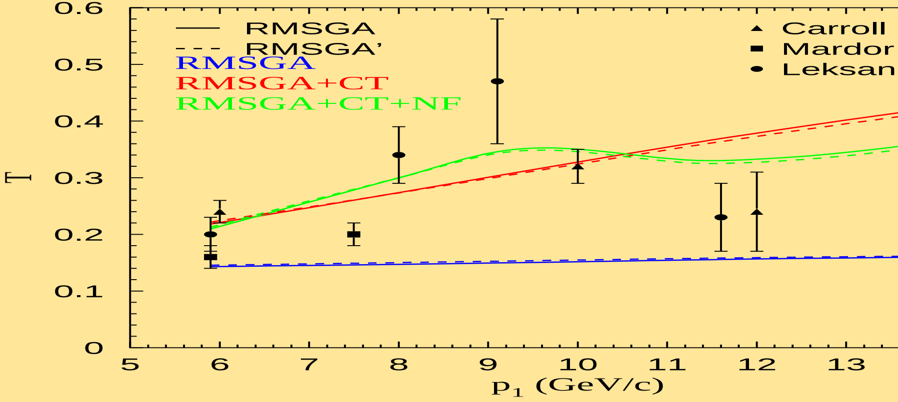
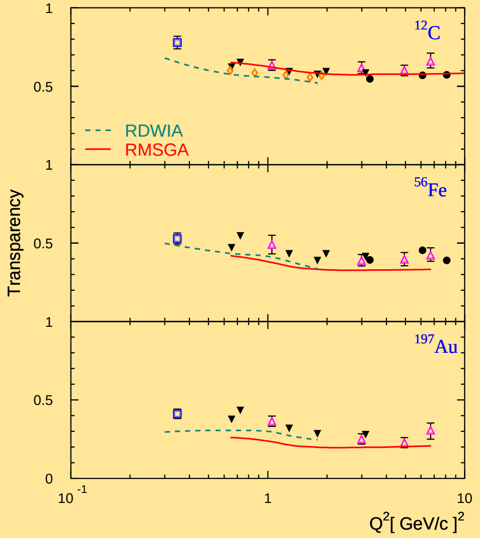


Parameterization of the CT effects compatible with pion production results!

The nuclear transparency from $A(e,e'p)$



- Calculations tend to underestimate the measured proton transparencies
- In the region of overlap: RMSGA and RDWIA predictions are not dramatically different !!
- Data from MIT, JLAB and SLAC
- CT effects are very small for $Q^2 \leq 10$ GeV^2

Second-order eikonal corrections for $A(e, e'p)$

- The eikonal approximation has a long and successful history
- One can compute so-called second-order eikonal corrections
- SOROMEA: Second Order Relativistic Optical Model Eikonal Approximation for the exclusive $A(e, e'p)$
- Unfactorized: not an issue in transparency calculations!
- Unfactorized: observables like "left-right" asymmetries can be computed

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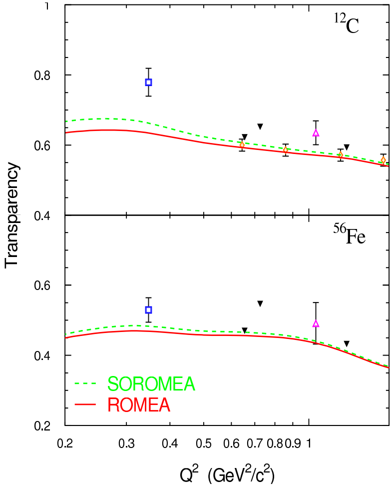

Second-order eikonal corrections to transparencies are very small !!

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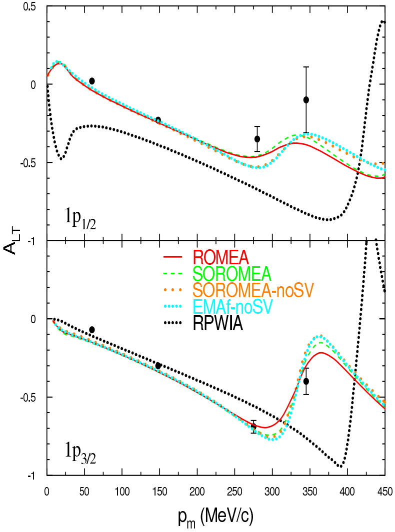

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Summary and outlook (I)

- recent times have seen a lot of theoretical activity in eikonal approaches to nuclear attenuation effects in exclusive $\gamma^{(*)}A \rightarrow B + \text{hadrons}$ *(Miller, Sargsian, Ciofi degli Atti, ...)*
- eikonal approach has enjoyed many successes in RIB physics *(Tostevin, Bertulani, ...)*
- RMSGA: “flexible” eikonal framework which can be used in combination with relativistic bound-state and continuum wave functions
- importance of implementing SRC in transparency calculations!
- the central short-range correlations make the nucleus more transparent for emission of fast pions and nucleons

Summary and outlook (II)

- $A(\gamma, \pi^- p)$: Nuclear transparencies from relativistic Glauber framework are larger than those from semi-classical models.
- CT and SRC exhibit a different "hard-scale" and A dependence **separation between hadronic and non-hadronic effects remains possible!**
- the $A(e, e'\pi^+)$ transparency results show deviations from traditional nuclear physics expectations AND are compatible with the educated estimates of how CT should be like !
- Robustness of the Glauber approximation:
  1. Semiclassical and RMSGA calculations provide similar pion transparencies
  2. Second-order eikonal corrections are small (even at low energies)
- JLAB at 12 GeV and JPARC (GSI?) ($(p, 2p)$) will provide the data to explore the crossover regime and establish the CT effect on a firm footing